# PUBLICATIONS

OF THE

# VASSAR COLLEGE OBSERVATORY

## No. 1

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MARY W. WHITNEY, Director

PUBLICATIONS OF THE VASSAR COLLEGE OBSERVATORY, No. 1.

MARY W. WHITNEY, *Director*.

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# CATALOGUE OF STARS

WITHIN

# ONE DEGREE OF THE NORTH POLE

AND

## OPTICAL DISTORTION OF THE HELSINGFORS ASTRO-PHOTOGRAPHIC TELESCOPE

DEDUCED FROM

# PHOTOGRAPHIC MEASURES

BY

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Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Faculty of Pure Science, Columbia University.

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## INTRODUCTION.

The Vassar College Observatory was established in 1865, when Vassar College was first opened to the public. It was built and equipped for purposes of instruction, the plan of founder and trustees not including scientific work for publication. The first director was Maria Mitchell, well known as a discoverer of comets, and as the recipient of a medal from the King of Denmark for the discovery of a comet in 1847. Professor Mitchell published a series of observations on the surface features of Jupiter and Saturn in *Silliman's Journal* and the *American Journal of Science.* Since 1890 an effort has been made to carry on a certain amount of observational work outside that involved in instruction, and this has been given mainly to minor planets and comets. Publications have been sent with more or less regularity to the current astronomical journals. The present paper is the first regular publication of the observatory.

The site of the observatory is on the grounds of Vassar College, about three miles east of the Hudson River, in Poughkeepsie, New York.

The adopted latitude of the observatory is 41° 41′ 18″. The adopted longitude of the observatory is $4^h$ $55^m$ $33^s.6$ west of Greenwich.

The latitude was obtained by Professor Mitchell in 1872, by the zenith telescope method. The telescope was loaned by the U. S. Coast Survey. The longitude was also obtained by Professor Mitchell, by electric telegraph connection with Harvard College Observatory in 1877. Clock signals were exchanged for one evening only, and there was no change of observers.

The observatory consists of a central dome, with wings to the south, east and north. The south wing contains a clock room and a class room, the east wing, the meridian room, the north wing the living rooms of director and assistant. In the dome, twenty-five feet in diameter, stands a twelve-inch equatorial with the usual equipment of eye-pieces and filar micrometer. The object glass of the equatorial was originally made by Fitz of New York, but was afterwards recut by Alvan Clark. The telescope was remounted in 1888 by Warner and Swasey of Cleveland, Ohio.

The meridian room contains a transit circle made by Young of Philadelphia, and afterward improved in its mounting by Fauth & Co., of Washington, D. C. The object glass of the transit telescope has an aperture of four inches. The clock room contains a Bond sidereal clock and a Bond chronograph; also a machine for measuring stellar photographs, made by the Repsold firm of Hamburg. As this publication treats only of work upon photographic plates, measured with the Repsold machine, it is not necessary that a more detailed account of the observatory equipment should be given at this time.

The Repsold measuring machine was the joint gift to the observatory of Miss Catherine Bruce and Mr. Frederick Thompson. It is of the pattern described by Scheiner in *Photographie der Gestirne*, p. 148. The additional details of its construction are given in the following pages.

The object glass of the equatorial is cut for visual purposes and we possess no correcting lens. It does not seem at present advisable, with the means and time at our disposal, to attempt to take photographs, but the measurement and reduction of photographs taken elsewhere fit in well with our other duties, and with this purpose in view the Repsold measuring machine was secured.

The first piece of reduction, based upon stellar photographs, was suggested by Professor Jacoby of Columbia University. He had in his possession twelve polar plates taken by Professor Donner of Helsingfors, Finland, partly measured and awaiting reduction. Miss Caroline E. Furness, assistant in the observatory, was placed in charge of the investigation.

We express our thanks to Professor Rees and Professor Jacoby of Columbia University, who have given us much aid and suggestion.

Our thanks are also due to our college trustee, Mr. Samuel Coykendall, through whose generosity this publication is made possible.

Miss Mary E. Tarbox and Miss Louise Ware, graduates of Vassar College, have rendered much assistance in the labor of computation.

We take this opportunity to give public expression to our thanks for the publications received from other observatories.

MARY W. WHITNEY,
VASSAR COLLEGE. *Director of Observatory.*

# TABLE OF CONTENTS.

*Catalogue of Stars within One Degree of the North Pole, and Optical Distortion of the Helsingfors Astro-photographic Telescope, deduced from Photographic Measures.*

I.

## Description of the Plates.

In September, 1895, at the request of Professor Harold Jacoby, of Columbia University, Professor Anders Donner, director of the Observatory at Helsingfors, Finland, made a series of photographs of the region of the North Pole. Professor Jacoby had two objects in view in securing these photographs; first, to obtain a catalogue of stars in the immediate vicinity of the pole; and second, to investigate by a new method * the optical distortion of the Helsingfors telescope. Twelve photographs were made in all; four with the telescope pointed at 90° declination, and the remaining eight with it pointed at 89°. The first four plates furnish material for a catalogue of stars within one degree of the pole, and their reduction forms the greater portion of the present paper. The discussion of optical distortion in Section X is based upon the results embodied in this catalogue. A reduction of the remaining eight plates, and a catalogue of the stars found on them are reserved for a later publication.

The photographs were taken in the following manner on plates provided with a reseau; the telescope was set for the desired right ascension and 89° declination, and a plate exposed. It was next moved 1° in declination, keeping the same right ascension, and a second plate exposed. The telescope was then moved 1° in declination a second time, still keeping the same right ascension, and a third plate exposed. For a second set of three plates the telescope was adjusted to a right ascension differing from the first by 45°, and the changes in declination just described were repeated, and similarly for a third and fourth set. The twelve plates taken together cover a circular area 2° in radius about the pole, with considerable overlapping. The four middle

* Astronomical Journal, Vol. XIII, p. 190.

plates contain the region within 1° of the pole. Within this limit they duplicate each other, but are taken with the telescope in different positions. Three exposures were made on each plate, $6^m$, $3^m$, and $20^s$ in duration, and between consecutive exposures the telescope was moved slightly in declination. Most of the stars were so faint that the $20^s$ images could not be measured; consequently they form no part of this reduction. The accompanying figure shows the overlapping of the twelve plates:

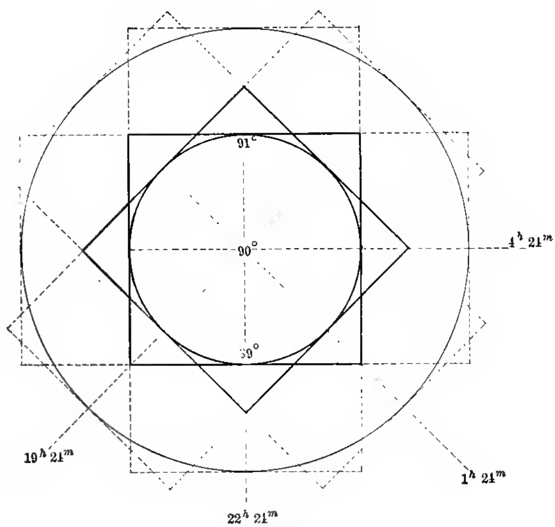


FIG. 1.

The following table taken from the Helsingfors records gives the data accompanying the exposures for the four central plates: Hereafter the plates will usually be designated by the Roman numeral in the last column.

TABLE I.

| Plate. 1895. | Obs. Barom. | Thermom. Att. R. | Thermom. Ext. R. | Length of Expos. | End of Expos. | α | δ | No. |
|---|---|---|---|---|---|---|---|---|
| | mm | ° | ° | m s | h m s | h m | ° ′ | |
| Sept. 16, No. 3 | Dr. 757.3 | +8.3 | +7.7 | 6 0<br>3 0<br>20 | 23 14 2<br>18 4<br>19 5 | 22 24 | 90 0 | I |
| Sept. 16, No. 6 | D. 756.6 | +8.0 | +7.2 | 6 0<br>3 0<br>20 | 0 27 59<br>31 35<br>32 36 | 1 24 | 90 0 | II |
| Sept. 18, No. 2 | Dr. 751.2 | +8.7 | +8.2 | 6 0<br>3 0<br>20 | 20 42 9<br>45 43<br>46 34 | 19 24 | 90 0 | III |
| Sept. 21, No. 14 | D. 764.5 | +2.5 | +2.1 | 6 0<br>3 0<br>20 | 3 18 34<br>22 12<br>23 2 | 4 24 | 90 0 | IV |

D. Donner.
Dr. Dreyer, assistant.

## II.

## Measurement of the Plates.

The star images were measured for the most part at the Columbia Observatory with the Repsold measuring machine No. 1, by Mrs. Herman S. Davis and Mrs. Annie Maclear Jacoby. A few were measured by me with the Repsold machine of the Vassar College Observatory. These measuring machines are of the form described in Scheiner's Photographie der Gestirne, p. 148. The reading microscope is furnished with two sets of movable micrometer threads at right angles to each other. In making the measurements at Columbia the horizontal screw was used throughout, since it was decided after trial that bisections made with vertical threads were more accurate than those made with horizontal threads.

The plate was oriented in the machine so that one set of reseau lines was parallel to the vertical micrometer threads, and the microscope adjusted so that ten revolutions were equal to the distance between two reseau lines, or $5^{mm}$. The screw-head was then set at $4^r.75$ and the carriage moved so as to place the parallel threads over the preceding corners of the reseau square. Micrometer settings was then made in the following order: 1° upper left hand corner, 2° lower left hand corner, 3° large image, 4° middle image, 5° small image, 6° lower right hand corner, 7° upper right hand corner. The last setting was about $14^r.75$. The screw was then turned forward half a revolution, to $15^r.25$, and the process just described was repeated in the reverse order, ending at $5^r.25$. The object of turning the screw half a revolution forward was to eliminate as far as possible the periodic errors, and to distribute the wear on the screw.

It was deemed necessary to observe the corners of each square, because the lines for the most part were too indistinct for accurate bisection. In some cases even the corners were invisible and this prevented the measurement of some of the stars.

Each coördinate was measured in two positions of the circle 180° apart, in order to eliminate personal errors entering into the bisection of the star image.

A specimen sheet showing the form of recording the settings will be found on page 8. The first two lines of the upper and lower sections contain the actual settings on the reseau lines and star images. In addition to the original record of the micrometer readings, each sheet contains the instrumental corrections, and all the steps leading to the determination of the final coördinates. Each image was reduced separately; in fact, the means were not taken until the final right ascension and polar distance were obtained.

## III.

## Instrumental Corrections.

1°. Screw Corrections.

The portion of the screw used lies between $5^r$ and $15^r$. The variations in pitch of this portion of the screw were determined by comparison with an accurate quarter millimeter scale, especially constructed for this purpose by Repsold, and the property of Professor Jacoby, who kindly lent it to Professor Whitney for determining the corrections of the Vassar screw. The method of comparison is described by Professor Jacoby in the *American Journal of Science*, 1896, Vol. 1, page 333. The following tables give the corrections to the readings of the horizontal screw in tenths of microns, or units of the fourth decimal place. These screw corrections, as will be seen from the specimen reduction sheet on page 8, were applied to the settings on the star-images only. Since the mean of the settings on each reseau line is very nearly $5^r$ or $15^r$, it was assumed that the screw corrections for the four separate settings on each line would nullify one another.

TABLE II.

| Micrometer. | Columbia, No. 1. | Vassar. |
|---|---|---|
| 5R | +0.0000 | +0.0000 |
| 6 | +0.0010 | +0.0028 |
| 7 | +0.0004 | +0.0038 |
| 8 | —0.0006 | +0.0034 |
| 9 | —0.0024 | +0.0018 |
| 10 | —0.0034 | +0.0000 |
| 11 | —0.0044 | —0.0006 |
| 12 | —0.0042 | —0.0018 |
| 13 | —0.0044 | —0.0028 |
| 14 | —0.0028 | —0.0030 |
| 15 | +0.0000 | +0.0000 |

2°. Reseau Corrections.

The reseau used, Gautier No. 9, has been investigated by Professor Donner, and the result published.* On pages 38–40 of his paper will be found two tables of corrections to the measured coördinates; one set for the $X$-coördinates, and one set for the $Y$-coördinates. They give the correction for the intersection of every vertical line with every horizontal line. The unit is a tenth micron. These corrections are applied in the following manner: Suppose $l$ and $l + 1$ are the numbers of two consecutive $X$-lines, and $l'$ and $l' + 1$ the numbers of two consecutive $Y$-lines, the four lines enclosing a square. Let $K_l$ be the correction for the line $l$, and $K_{l+1}$ the correction for the line $l + 1$. Then $K_l$ is the mean of the tabular values for the intersection of the $X$-line with the two $Y$-lines $l'$ and $l' + 1$; $K_{l+1}$ is the mean of the tabular values for the intersection of the $X$-line $l + 1$ with the same two $Y$-lines. In other words, we find the correction for the middle of the side of the square.

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* Acta Societatis Scientiarum Fennicæ. Tome XXI, Nr. 8. Détermination des Constantes nécessaires pour la Réduction des Clichés pris à Helsingfors.

3°. Error of Runs.

The error of runs for each square can be obtained directly from the settings on the reseau lines. The method of correcting for it will be given in the derivation of the general formula for finding the star's coördinate from the measurements.

4°. Method of Obtaining the Corrected Cordinates.

Let $l$ = number of lower reseau line,
$l + 1$ = number of upper reseau line,
$D$ = difference between the reading of the lower line and the reading of the star image, always positive.
$Q$ = error of runs expressed in units of the fourth decimal place,
$R$ = difference between the readings of the upper and lower reseau lines, always positive, $= 10.0000 \pm Q$,
$K_l$ = correction to position of lower reseau line taken from Donner's table,
$K_{l+1}$ = correction for upper reseau line,
$X, Y$ = final coördinates.

Then if we measure the coördinates from the line 0, *e. g.*, $x$-coördinate,

$$X = 5l + K_l + \frac{5D}{R} + (K_{l+1} - K_l)\frac{D}{R}.$$

Since $R$ will not differ much from $10^r$ it will be sufficient if in place of $\frac{1}{R}$ we use $\frac{1}{10} \mp \frac{Q}{100}$. The formula then becomes

$$X = 5l + \left\{ K_l + \frac{1}{2} D + (K_{l+1} - K_l)\frac{D}{10} \mp \frac{1}{2} Q \frac{D}{10} \right\}.$$

In case we wish to measure the coördinates from the central reseau lines we should use for $5l$, $5(l-14)$ and $5(l-43)$.

This method of recording and reducing is due to Professor Jacoby.

The specimen sheet appended shows in detail the method of obtaining the final coördinate. The screw correction has been incorporated with the general sum in order to shorten the process.

The succeeding pages contain Table III, the table of corrected coördinates. It includes the coördinates, direct, reversed and mean, for both the big and middle images of every star on the four plates.

## POLAR PLATE: HELS. SEPT. 16, '95. NO. 3.

| Line. | | December 2, 1896: *X* Direct. | | | | Line. | |
|---|---|---|---|---|---|---|---|
| No. | Micr. | Big Image. | Sc. | Mid. Image. | Sc. | No. | Micr. |
| 20 | 4.765<br>.754<br>5.250<br>.264 | 9.152<br>9.660 | +15<br>+ 6 | 9.142<br>9.639 | +15<br>+ 6 | 21 | 14.738<br>.748<br>15.248<br>.244 |
| | 5.0082 | 9.4060 | +10 | 9.3905 | +10 | | 14.9945 |
| Low Line Read.<br>Diff. = *D*. | | 5.0082<br>4.3978 | | 5.0082<br>4.3823 | | | 5.0082<br>9.9863 |
| | | | | | | $Q =$ | —137 |
| ½ *D* | | + 2.1989 | + 5 | + 2.1912 | + 5 | | |
| $K_l$ | | + 6 | | + 6 | | $K_{l+1}$ | +5 |
| | | | | | | $K_l =$ | +6 |
| $(K_{l+1} - K_l)\frac{D}{10}$ | | + 0 | | + 0 | | Diff. = | —1 |
| $-\frac{1}{2} Q \frac{D}{10}$ | | + 30 | | + 30 | | | |
| Screw Corr'n | | + 5 | | + 5 | | | |
| Sum | | + 2.2030 | | + 2.1953 | | | |
| Lower Line, mm. | | —30. | | —30. | | | |
| Star's Coord. | | —32.2030 | | —32.1953 | | | |

| Line. | | November 18, 1896: *X* reversed. | | | | Line. | |
|---|---|---|---|---|---|---|---|
| No. | Micr. | Big Image. | Sc. | Mid. Image. | Sc. | No. | Micr. |
| 21 | 4.776<br>.737<br>5.202<br>.232 | 10.364<br>10.812 | — 2<br>— 5 | 10.375<br>10.842 | — 2<br>— 5 | 20 | 14.750<br>.773<br>15.229<br>.201 |
| | 4.9868 | 10.5880 | — 4 | 10.6085 | — 4 | | 14.9882 |
| Low Line Read.<br>Diff. = *D*. | | 14.9882<br>4.4002 | | 14.9882<br>4.3797 | | | 4.9868<br>10.0014 |
| | | | | | | $Q = +$ | 14 |
| ½ *D* | | + 2.2001 | + 2 | + 2.1898 | + 2 | | |
| $K_l$ | | + 6 | | + 6 | | $K_{l+1}$ | +5 |
| | | | | | | $K_l =$ | +6 |
| $(K_{l+1} - K_l)\frac{D}{10}$ | | + 0 | | + 0 | | Diff. = | —1 |
| $-\frac{1}{2} Q \frac{D}{10}$ | | — 3 | | — 3 | | | |
| Screw Corr'n | | + 2 | | + 2 | | | |
| Sum | | + 2.2006 | | + 2.1903 | | | |
| Lower Line. mm. | | —30. | | —30. | | | |
| Star's Coord. | | —32.2006 | | —32.1903 | | | |
| Mean *X* | | —32.2018 | | —32.1928 | | | |

## STAR 43. MEASURED BY FURNESS.

| Line. | | December 3, 1896: *Y* Direct. | | | | Line. | |
|---|---|---|---|---|---|---|---|
| No. | Micr. | Big Image. | Sc. | Mid. Image. | Sc. | No. | Micr. |
| 44 | 4.752 | 8.219 | +31 | 8.925 | +20 | 45 | 14.733 |
| | .750 | 8.740 | +22 | 9.446 | +10 | | .741 |
| | 5.278 | | | | | | 15.270 |
| | .279 | | | | | | .250 |
| | 5.0148 | 8.4795 | +26 | 9.1855 | +15 | | 14.9985 |
| Low Line Read. | | 5.0148 | | 5.0148 | | | 5.0148 |
| Diff. = *D*. | | 3.4647 | | 4.1707 | | | 9.9837 |
| | | | | | | $Q =$ | −163 |
| ½ *D* | | +1.7324 | +13 | +2.0854 | + 8 | $K_{l+1}$ | +0 |
| $K_l$ | | − 9 | | − 9 | | $K_l =$ | −9 |
| $(K_{l+1} - K_l)\frac{D}{10}$ | | + 3 | | + 4 | | Diff. = | +9 |
| $-\frac{1}{2} Q \frac{D}{10}$ | | + 28 | | + 34 | | | |
| Screw Corr'n | | + 13 | | + 8 | | | |
| Sum | | +1.7359 | | +2.0891 | | | |
| Lower Line, mm. | | +5. | | +5. | | | |
| Star's Coord. | | +6.7359 | | +7.0891 | | | |
| **Line.** | | **November 20, 1896: *Y* Reversed.** | | | | **Line.** | |
| No. | Micr. | Big Image. | Sc. | Mid. Image. | Sc. | No. | Micr. |
| 45 | 4.770 | 11.299 | −11 | 10.590 | − 4 | 44 | 14.752 |
| | .740 | 11.790 | −20 | 11.072 | − 7 | | .770 |
| | 5.241 | | | | | | 15.263 |
| | .260 | | | | | | .237 |
| | 5.0028 | 11.5445 | −16 | 10.8310 | − 6 | | 15.0055 |
| Low Line Read. | | 15.0055 | | 15.0055 | | | 5.0028 |
| Diff. = *D*. | | 3.4610 | | 4.1745 | | | 10.0027 |
| | | | | | | $Q =$ | +27 |
| ½ *D*. | | + 1.7305 | + 8 | + 2.0872 | + 3 | $K_{l+1}$ | +0 |
| $K_l$ | | − 9 | | − 9 | | $K_l =$ | −9 |
| $(K_{l+1} - K_l)\frac{D}{10}$ | | + 3 | | + 4 | | Diff. = | +9 |
| $-\frac{1}{2} Q \frac{D}{10}$ | | − 5 | | − 6 | | | |
| Screw Corr'n | | + 8 | | + 3 | | | |
| Sum | | + 1.7302 | | + 2.0864 | | | |
| Lower Line, mm. | | + 5. | | + 5. | | | |
| Star's Coord. | | + 6.7302 | | + 7.0864 | | | |
| Mean *Y* | | + 6.7330 | | + 7.0878 | | | |

TABLE III.—CORRECTED COÖRDINATES. PLATE I.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | +22.9453 | .9481 | +22.9467 | —50.8083 | .8062 | —50.8072 |
| 2 | + 5.3844 | .3885 | + 5.3864 | — 9.7128 | .7157 | — 9.7142 |
| 3 | +24.9746 | .9720 | +24.9733 | —44 4794 | .4753 | —44.4774 |
| 4 | +35.4435 | .4493 | +35.4464 | —32.5757 | .5693 | —32.5725 |
| 6 | +44.1476 | .1551 | +44.1514 | —25.9000 | .9007 | —25.9004 |
| 7 | +31.3441 | .3390 | +31.3416 | —17.7044 | .7070 | —17.7057 |
| 8 | +31.3513 | .3477 | +31.3495 | —16.3648 | .3659 | —16.3654 |
| 9 | +23.3213 | .3258 | +23.3236 | —11.3905 | .3858 | —11.3882 |
| 10 | +45.0110 | .0067 | +45.0088 | —20.5572 | .5529 | —20.5550 |
| 11 | +21.2718 | .2685 | +21.2702 | — 7.7350 | .7347 | —11.7348 |
| 12 | +47.7341 | .7311 | +47.7326 | — 6.6388 | .6385 | — 6.6386 |
| 13 | +43.1034 | .1191 | +43.1112 | — 5.2224 | .2265 | — 5.2244 |
| 14 | +59.5247 | .5303 | +59.5275 | — 2.8530 | .8522 | — 2.8526 |
| 15 | +52.8485 | .8508 | +52.8496 | — 1.8472 | .8619 | — 1.8546 |
| 16 | +58.7837 | .7820 | +58.7828 | + 1.7625 | .7564 | + 1.7595 |
| 17 | +54.0363 | .0407 | +54.0385 | + 2.9028 | .8930 | + 2.8979 |
| 18 | +19.9844 | .9886 | +19.9865 | + 2.7839 | .7842 | + 2.7840 |
| 19 | +38.4077 | .4091 | +38.4084 | + 6.7411 | .7464 | + 6.7438 |
| 20 | +34.3779 | .3802 | +34.3790 | +11.3692 | .3575 | +11.3634 |
| 21 | +27.9325 | .9345 | +27.9335 | +14.8407 | .8459 | +14.8433 |
| 22 | +46.2865 | .2915 | +46.2890 | +29.2586 | .2564 | +29.2575 |
| 23 | +10.7241 | .7252 | +10.7246 | + 8.8621 | .8614 | + 8.8618 |
| 24 | +47.4451 | .4495 | +47.4474 | +33.1202 | .1318 | +33.1260 |
| 25 | +14.7758 | .7806 | +14.7782 | +21.9611 | .9610 | +21.9610 |
| 26 | + 8.1195 | .1169 | + 8.1182 | +21.4081 | .4070 | +21.4076 |
| 27 | + 1.3331 | .3257 | + 1.3294 | +35.1691 | .1702 | +35.1696 |
| 28 | — 4.6425 | .6338 | — 4.6382 | +39.8213 | .8227 | +39.8220 |
| 29 | — 9.6857 | .6866 | — 9.6862 | +38.2426 | .2424 | +38.2425 |
| 30 | — 7.7410 | .7415 | — 7.7412 | +27.6632 | .6615 | +27.6624 |
| 31 | —18.1615 | .1605 | —18.1610 | +42.9125 | .9106 | +42.9116 |
| 32 | —19.8304 | .8352 | —19.8328 | +30.3051 | .3055 | +30.3053 |
| 33 | —24.2575 | .2595 | —25.2585 | +36.8582 | .8603 | +36.8592 |
| 34 | —24.3796 | .3800 | —24.3798 | +36.9228 | .9349 | +36.9288 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE I.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 35 | —24.0960 | .0995 | —24.0978 | +31.4850 | .4885 | +31.4868 |
| 38 | —23.0990 | .0978 | —23.0984 | +17.8260 | .8167 | +17.8214 |
| 39 | —31.7337 | .7308 | —31.7322 | +20.7775 | .7733 | +20.7754 |
| 40 | —35.7056 | .7096 | —35.7076 | +21.9363 | .9363 | +21.9363 |
| 41 | —22.4889 | .4895 | —22.4892 | + 8.7226 | .7145 | + 8.7186 |
| 42 | —19.6396 | .6389 | —19.6392 | + 4.9837 | .9840 | + 4.9838 |
| 43 | —32.2030 | .2006 | —32.2018 | + 6.7359 | .7302 | + 6.7330 |
| 44 | —47.5636 | .5657 | —47.5646 | + 4.0003 | .9911 | + 3.9957 |
| 45 | —12.7930 | .7933 | —12.7932 | + 0.0078 | .0051 | + 0.0064 |
| 46 | —20.4808 | .4786 | —20.4796 | — 3.8991 | .8944 | — 3.8968 |
| 47 | —52.1609 | .1574 | —52.1592 | —13.5107 | .5116 | —13.5112 |
| 48 | — 5.8905 | .8946 | — 5.8926 | — 0.1050 | .1047 | — 0.1048 |
| 49 | — 6.0505 | .0540 | — 6.0522 | — 0.2313 | .2348 | — 0.2330 |
| 50 | —47.1179 | .1118 | —47.1148 | —16.2131 | .2155 | —16.2143 |
| 51 | —26.2338 | .2347 | —26.2342 | —12.8866 | .8903 | —12.8884 |
| 52 | —36.9373 | .9353 | —36.9363 | —22.9566 | .9528 | —22.9547 |
| 53 | —17.4309 | .4344 | —17.4326 | — 9.7622 | .7598 | — 9.7610 |
| 54 | —32.5714 | .5758 | —32.5736 | —23.5097 | .5046 | —23.5072 |
| 55 | — 6.6942 | .6922 | — 6.6932 | — 4.2510 | .2476 | — 4.2493 |
| 56 | —19.6199 | .6221 | —19.6210 | —17.0983 | .0997 | —17.0990 |
| 57 | —12.8857 | .8816 | —12.8836 | —11.1209 | .1226 | —11.1218 |
| 58 | —14.6145 | .6116 | —14.6130 | —14.8666 | .8719 | —14.8692 |
| 59 | —13.6072 | .6000 | —13.6036 | —20.6847 | .6791 | —20.6819 |
| 60 | — 6.9856 | .9834 | — 6.9845 | —11.1557 | .1488 | —11.1522 |
| 61 | —10.8260 | .8248 | —10.8254 | —22.7863 | .7891 | —22.7877 |
| 62 | — 9.1502 | .1507 | — 9.1504 | —19.3626 | .3601 | —19.3614 |
| 63 | —19.0203 | .0235 | —19.0219 | —49.5376 | .5436 | —49.5406 |
| 64 | — 5.9269 | .9238 | — 5.9248 | —17.5011 | .5049 | —17.5030 |
| 65 | + 8.6687 | .6674 | + 8.6680 | —46.0112 | .0084 | —46.0098 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE I.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | +22.9457 | .9469 | +22.9463 | —50.4622 | .4633 | —50.4628 |
| 2 | + 5.3914 | .3870 | + 5.3892 | — 9.3647 | .3662 | — 9.3654 |
| 3 | +24.9746 | .9651 | +24.9698 | —44.1294 | .1266 | —44.1280 |
| 4 | +35.4477 | .4451 | +35.4464 | —32.2295 | .2312 | —32.2304 |
| 6 | +44.1471 | .1589 | +44.1530 | —25.5588 | .5669 | —25.5628 |
| 7 | +31.3459 | .3370 | +31.3414 | —17.3766 | .3724 | —17.3745 |
| 8 | +31.3431 | .3402 | +31.3416 | —16.0263 | .0259 | —16.0261 |
| 9 | +23.3245 | .3262 | +23.3254 | —11.0444 | .0438 | —11.0441 |
| 10 | +45.0095 | .0092 | +45.0094 | —20.2230 | .2150 | —20.2190 |
| 11 | +21.2705 | .2708 | +21.2706 | — 7.3943 | .3913 | — 7.3928 |
| 12 | +47.7374 | .7423 | +47.7398 | — 6.3030 | .3051 | — 6.3040 |
| 13 | +43.1010 | .1043 | +43.1026 | — 4.8896 | .8964 | — 4.8930 |
| 14 | +59.5325 | .5367 | +59.5346 | — 2.5175 | .5183 | — 2.5179 |
| 15 | +52.8525 | .8513 | +52.8519 | — 1.5268 | .5269 | — 1.5268 |
| 16 | +58.7840 | .7792 | +58.7816 | + 2.0911 | .0928 | + 2.0920 |
| 17 | +54.0368 | .0472 | +54.0420 | + 3.2383 | .2299 | + 3.2341 |
| 18 | +19.9924 | .9889 | +19.9906 | + 3.1240 | .1263 | + 3.1252 |
| 19 | +38.4134 | .4088 | +38.4111 | + 7.0778 | .0783 | + 7.0780 |
| 20 | +34.3885 | .3762 | +34.3824 | +11.7005 | .7049 | +11.7027 |
| 21 | +27.9405 | .9407 | +27.9406 | +15.1763 | .1793 | +15.1778 |
| 22 | +46.3023 | .3025 | +46.3024 | +29.6019 | .5974 | +29.5995 |
| 23 | +10.7264 | .7283 | +10.7274 | + 9.2029 | .2047 | + 9.2038 |
| 24 | +47.4523 | .4495 | +47.4509 | +33.4660 | .4718 | +33.4689 |
| 25 | +14.7862 | .7903 | +14.7882 | +22.3027 | .3051 | +22.3039 |
| 26 | + 8.1301 | .1256 | + 8.1278 | +21.7507 | .7551 | +21.7529 |
| 27 | + 1.3337 | .3323 | + 1.3330 | +35.5142 | .5185 | +35.5164 |
| 28 | — 4.6283 | .6248 | — 4.6266 | +40.1664 | .1654 | +40.1659 |
| 29 | — 9.6765 | .6746 | — 9.6756 | +38.5882 | .5924 | +38.5903 |
| 30 | — 7.7242 | .7224 | — 7.7233 | +28.0109 | .0077 | +28.0093 |
| 31 | —18.1630 | .1570 | —18.1600 | +43.2654 | .2654 | +43.2654 |
| 32 | —19.8218 | .8279 | —19.8248 | +30.6590 | .6546 | +30.6568 |
| 33 | —24.2545 | .2578 | —24.2562 | +37.2014 | .2138 | +37.2076 |
| 34 | —24.3774 | .3770 | —24.3772 | +37.2744 | .2817 | +37.2780 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE I.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 35 | —24.0933 | .0991 | —24.0962 | +31.8348 | .8386 | +31.8367 |
| 38 | —23.0900 | .0895 | —23.0898 | +18.1781 | .1702 | +18.1742 |
| 39 | —31.7220 | .7212 | —31.7216 | +21.1174 | .1019 | +21.1096 |
| 40 | —35.7061 | .7109 | —35.7085 | +22.2888 | .2804 | +22.2846 |
| 41 | —22.4875 | .4820 | —22.4848 | + 9.0809 | .0721 | + 9.0765 |
| 42 | —19.6368 | .6393 | —19.6380 | + 5.3342 | .3373 | + 5.3358 |
| 43 | —32.1953 | .1903 | —32.1928 | + 7.0891 | .0864 | + 7.0878 |
| 44 | —47.5546 | .5577 | —47.5562 | + 4.3522 | .3293 | + 4.3408 |
| 45 | —12.7915 | .7916 | —12.7916 | + 0.3595 | .3586 | + 0.3590 |
| 46 | —20.4775 | .4776 | —20.4776 | — 3.5418 | .5424 | — 3.5421 |
| 47 | —52.1630 | .1530 | —52.1580 | —13.1664 | .1653 | —13.1658 |
| 48 | — 5.8889 | .8894 | — 5.8892 | + 0.2455 | .2429 | + 0.2442 |
| 49 | — 6.0463 | .0505 | — 6.0484 | + 0.1126 | .1132 | + 0.1129 |
| 50 | —47.1164 | .1092 | —47.1128 | —15.8591 | .8587 | —15.8589 |
| 51 | —26.2308 | .2357 | —26.2332 | —12.5344 | .5359 | —12.5352 |
| 52 | —36.9319 | .9300 | —36.9310 | —22.6044 | .6066 | —22.6055 |
| 53 | —17.4261 | .4334 | —17.4298 | — 9.4071 | .4107 | — 9.4089 |
| 54 | —32.5761 | .5735 | —32.5748 | —23.1595 | .1590 | —23.1592 |
| 55 | — 6.6944 | .6967 | — 6.6956 | — 3.8973 | .9004 | — 3.8988 |
| 56 | —19.6157 | .6201 | —19.6179 | —16.7459 | .7436 | —16.7448 |
| 57 | —12.8827 | .8712 | —12.8769 | —10.7789 | .7833 | —10.7811 |
| 58 | —14.6118 | .6109 | —14.6114 | —14.5141 | .5129 | —14.5135 |
| 59 | —13.6152 | .6015 | —13.6084 | —20.3245 | .3192 | —20.3218 |
| 60 | — 6.9868 | .9798 | — 6.9833 | —10.8071 | .7989 | —10.8030 |
| 61 | —10.8260 | .8270 | —10.8265 | —22.4379 | .4423 | —22.4401 |
| 62 | — 9.1504 | .1510 | — 9.1507 | —19.0140 | .0119 | —19.0130 |
| 63 | —19.0226 | .0287 | —19.0256 | —49.1951 | .1936 | —49.1944 |
| 64 | — 5.9264 | .9271 | — 5.9268 | —17.1514 | .1527 | —17.1520 |
| 65 | + 8.6654 | .6670 | + 8.6662 | —45.6624 | .6621 | —45.6622 |

TABLE III.—(*Cont.*). CORRECTED COÖRDINATES. PLATE II.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | —19.6722 | .6754 | —19.6738 | —52.2321 | .2267 | —52.2294 |
| 2 | — 2.9149 | .9149 | — 2.9149 | —10.8103 | .8153 | —10.8128 |
| 3 | —13.7549 | .7517 | —13.7533 | —49.2109 | .2070 | —49.2090 |
| 4 | + 2.0730 | .0677 | + 2.0704 | —48.2452 | .2401 | —48.2426 |
| 5 | +13.3367 | .3392 | +13.3380 | —54.4698 | .4753 | —54.4726 |
| 6 | +12.9355 | .9357 | +12.9356 | —49.6995 | .7006 | —49.7000 |
| 7 | + 9.7075 | .7008 | + 9.7042 | —34.8477 | .8472 | —34.8474 |
| 8 | +10.6696 | .6668 | +10.6682 | —33.9065 | .9013 | —33.9039 |
| 9 | + 8.5342 | .5321 | + 8.5332 | —24.7039 | .7031 | —24.7035 |
| 10 | +17.3322 | .3345 | +17.3334 | —46.5470 | .5531 | —46.5505 |
| 11 | + 9.6825 | .6869 | + 9.6847 | —20.6904 | .6824 | —20.6864 |
| 12 | +29.1205 | .1183 | +29.1194 | —38.6722 | .6676 | —38.6699 |
| 13 | +26.8663 | .8633 | +26.8648 | —34.3931 | .3998 | —34.3964 |
| 14 | +40.1337 | .1273 | +40.1305 | —44.3621 | .3518 | —44.3570 |
| 15 | +36.1183 | .1181 | +36.1182 | —38.9181 | .9147 | —38.9164 |
| 16 | +42.8825 | .8710 | +42.8768 | —40.5763 | .5748 | —40.5756 |
| 18 | +16.2412 | .2347 | +16.2380 | —12.3442 | .3442 | —12.3442 |
| 19 | +32.0386 | .0409 | +32.0398 | —22.6153 | .6184 | —22.6168 |
| 21 | +30.3795 | .3881 | +30.3838 | — 9.4727 | .4729 | — 9.4728 |
| 22 | +53.5550 | .5531 | +53.5540 | —12.3302 | .3309 | —12.3306 |
| 23 | +14.0156 | .0159 | +14.0158 | — 1.4987 | .4921 | — 1.4954 |
| 24 | +57.1244 | .1235 | +57.1240 | —10.4147 | .4156 | —10.4152 |
| 25 | +26.1679 | .1687 | +26.1683 | + 4.8716 | .8695 | + 4.8706 |
| 26 | +21.0775 | .0767 | +21.0771 | + 9.2026 | .2100 | + 9.2063 |
| 27 | +26.0450 | .0480 | +26.0465 | +23.7134 | .7149 | +23.7142 |
| 28 | +25.1262 | .1239 | +25.1250 | +31.2233 | .2350 | +31.2292 |
| 29 | +20.4485 | .4544 | +20.4514 | +33.6936 | .6927 | +33.6932 |
| 30 | +14.3306 | .3343 | +14.3324 | +24.8473 | .8454 | +24.8464 |
| 31 | +17.7888 | .7866 | +17.7877 | +42.9988 | .0038 | +43.0013 |
| 32 | + 7.6611 | .6619 | + 7.6615 | +35.2929 | .2878 | +35.2904 |
| 33 | + 9.1945 | .1935 | + 9.1940 | +43.0544 | .0497 | +43.0520 |
| 34 | + 9.1485 | .1499 | + 9.1492 | +43.1841 | .1840 | +43.1840 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE II.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 35 | + 5.4983 | .5003 | + 5.4993 | +39.1522 | .1495 | +39.1508 |
| 36 | + 2.2747 | .2695 | + 2.2721 | +15.7789 | .7828 | +15.7808 |
| 37 | + 1.0478 | .0460 | + 1.0469 | +14.9990 | .9954 | +14.9972 |
| 38 | — 3.4856 | .4909 | — 3.4882 | +28.8050 | .8140 | +28.8095 |
| 39 | — 7.4851 | .4858 | — 7.4854 | +36.9957 | .9978 | +36.9968 |
| 40 | — 9.4610 | .4629 | — 9.4620 | +40.6423 | .6505 | +40.6464 |
| 41 | — 9.5169 | .5147 | — 9.5158 | +21.9523 | .9548 | +21.9536 |
| 42 | —10.1503 | .1478 | —10.1490 | +17.2989 | .2938 | +17.2964 |
| 43 | —17.7676 | .7734 | —17.7705 | +27.4390 | .4441 | +27.4416 |
| 44 | —30.5523 | .5473 | —30.5498 | +36.3914 | .3875 | +36.3894 |
| 45 | — 8.8399 | .8456 | — 8.8428 | + 8.9350 | .9397 | + 8.9374 |
| 46 | —17.0272 | .0235 | —17.0254 | +11.6352 | .6376 | +11.6364 |
| 47 | —46.1929 | .1950 | —46.1940 | +27.3002 | .3040 | +27.3021 |
| 48 | — 4.0621 | .0623 | — 4.0622 | + 3.9714 | .9688 | + 3.9701 |
| 49 | — 4.2688 | .2622 | — 4.2655 | + 3.9970 | .9938 | + 3.9954 |
| 50 | —44.5502 | .5581 | —44.5542 | +21.8303 | .8312 | +21.8308 |
| 51 | —27.4634 | .4648 | —27.4641 | + 9.3753 | .3792 | + 9.3772 |
| 52 | —42.1561 | .1596 | —42.1578 | + 9.8571 | .8541 | + 9.8556 |
| 53 | —19.0463 | .0426 | —19.0444 | + 5.3436 | .3415 | + 5.3426 |
| 54 | —39.4752 | .4711 | —39.4732 | + 6.3817 | .3772 | + 6.3794 |
| 56 | —25.7946 | .7925 | —25.7936 | + 1.7060 | .7074 | + 1.7067 |
| 57 | —16.8067 | .8082 | —16.8074 | + 1.1510 | .1526 | + 1.1518 |
| 58 | —20.6754 | .6714 | —20.6734 | — 0.2687 | .2752 | — 0.2720 |
| 60 | —12.6714 | .6683 | —12.6698 | — 3.0427 | .0465 | — 3.0446 |
| 61 | —23.6287 | .6254 | —23.6270 | — 8.5265 | .5306 | — 8.5286 |
| 62 | —20.0142 | .0125 | —20.0134 | — 7.3133 | .3099 | — 7.3116 |
| 63 | —48.3696 | .3714 | —48.3705 | —21.6059 | .5957 | —21.6008 |
| 64 | —16.4216 | .4256 | —16.4236 | — 8.2829 | .2755 | — 8.2792 |
| 65 | —26.3480 | .3452 | —26.3466 | —38.7338 | .7305 | —38.7322 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE II.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | —19.6819 | .6807 | —19.6813 | —51.8459 | .8429 | —51.8444 |
| 2 | — 2.9289 | .9277 | — 2.9283 | —10.4194 | .4244 | —10.4219 |
| 3 | —13.7549 | .7617 | —13.7583 | —48.8222 | .8225 | —48.8224 |
| 4 | + 2.0675 | .0576 | + 2.0626 | —47.8510 | .8485 | —47.8498 |
| 5 | +13.3322 | .3284 | +13.3303 | —54.0749 | .0832 | —54.0790 |
| 6 | +12.9340 | .9299 | +12.9320 | —49.3208 | .3190 | —49.3199 |
| 7 | + 9.6978 | .6910 | + 9.6944 | —34.4527 | .4553 | —34.4540 |
| 8 | +10.6586 | .6636 | +10.6611 | —33.5084 | .5108 | —33.5096 |
| 9 | + 8.5275 | .5231 | + 8.5253 | —24.3142 | .3107 | —24.3124 |
| 10 | +17.3244 | .3237 | +17.3240 | —46.1605 | .1622 | —46.1614 |
| 11 | + 9.6797 | .6764 | + 9.6780 | —20.2903 | .2945 | —20.2924 |
| 12 | +29.1192 | .1116 | +29 1154 | —38.2845 | .2796 | —38.2820 |
| 13 | +26.8483 | .8566 | +26.8524 | —33.9977 | .0040 | —34.0008 |
| 14 | +40.1204 | .1166 | +40.1185 | —43.9662 | .9672 | —43.9667 |
| 15 | +36.1131 | .1198 | +36.1164 | —38.5266 | .5280 | —38.5273 |
| 16 | +42.8584 | .8610 | +42.8597 | —40.1702 | .1700 | —40.1701 |
| 18 | +16.2284 | .2277 | +16.2280 | —11.9563 | .9520 | —11.9542 |
| 19 | +32.0258 | .0246 | +32.0252 | —22.2221 | .2226 | —22.2224 |
| 21 | +30.3745 | .3781 | +30.3763 | — 9.0853 | .0811 | — 9.0832 |
| 22 | +53.5429 | .5405 | +53.5418 | —11.9430 | .9419 | —11.9424 |
| 23 | +14.0034 | .0046 | +14.0040 | — 1.1102 | .1012 | — 1.1057 |
| 24 | +57.1119 | .1068 | +57.1094 | —10.0175 | .0178 | —10.0176 |
| 25 | +26.1563 | .1599 | +26.1581 | + 5.2628 | .2611 | + 5.2620 |
| 26 | +21.0685 | .0667 | +21.0676 | + 9.5909 | .5958 | + 9.5934 |
| 27 | +26.0320 | .0341 | +26.0330 | +24.1101 | .1136 | +24.1118 |
| 28 | +25.1196 | .1146 | +25.1171 | +31.6126 | .6230 | +31.6178 |
| 29 | +20.4415 | .4417 | +20.4416 | +34.0871 | .0904 | +34.0888 |
| 30 | +14.3201 | .3248 | +14.3224 | +25.2374 | .2329 | +25.2352 |
| 31 | +17.7798 | .7788 | +17.7793 | +43.3888 | .3827 | +43.3858 |
| 32 | + 7.6504 | .6446 | + 7.6475 | +35.6858 | .6792 | +35.6825 |
| 33 | + 9.1837 | .1830 | + 9.1834 | +43.4445 | .4448 | +43.4446 |
| 34 | + 9.1365 | .1394 | + 9.1380 | +43.5738 | .5680 | +43.5709 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE II.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 35 | + 5.4868 | .4882 | + 5.4875 | +39.5398 | .5381 | +39.5390 |
| 36 | + 2.2625 | .2576 | + 2.2600 | +16.1676 | .1725 | +16.1700 |
| 37 | + 1.0223 | .0110 | + 1.0166 | +15.3879 | .3878 | +15.3878 |
| 38 | — 3.4964 | .4988 | — 3.4976 | +29.2004 | .2045 | +29.2024 |
| 39 | — 7.4944 | .4991 | — 7.4968 | +37.3764 | .3753 | +37.3758 |
| 40 | — 9.4699 | .4667 | — 9.4683 | +41.0273 | .0417 | +41.0345 |
| 41 | — 9.5271 | .5284 | — 9.5278 | +22.3445 | .3474 | +22.3460 |
| 42 | —10.1631 | .1571 | —10.1601 | +17.6895 | .6812 | +17.6854 |
| 43 | —17.7791 | .7764 | —17.7778 | +27.8362 | .8309 | +27.8336 |
| 44 | —30.5611 | .5525 | —30.5568 | +36.7742 | .7723 | +36.7732 |
| 45 | — 8.8439 | .8503 | — 8.8471 | + 9.3256 | .3285 | + 9.3270 |
| 46 | —17.0385 | .0412 | —17.0398 | +12.0241 | .0306 | +12.0274 |
| 47 | —46.2009 | .2027 | —46.2018 | +27.6878 | .6869 | +27.6874 |
| 48 | — 4.0745 | .0768 | — 4.0756 | + 4.3608 | .3620 | + 4.3614 |
| 49 | — 4.2815 | .2787 | — 4.2801 | + 4.3841 | .3835 | + 4.3838 |
| 50 | —44.5634 | .5651 | —44.5642 | +22.2167 | .2185 | +22.2176 |
| 51 | —27.4762 | .4762 | —27.4762 | + 9.7622 | .7704 | + 9.7663 |
| 52 | —42.1659 | .1746 | —42.1702 | +10.2370 | .2390 | +10.2380 |
| 53 | —19.0580 | .0557 | —19.0568 | + 5.7317 | .7279 | + 5.7298 |
| 54 | —39.4882 | .4848 | —39.4865 | + 6.7667 | .7657 | + 6.7662 |
| 56 | —25.8048 | .8031 | —25.8040 | + 2.0992 | .0994 | + 2.0993 |
| 57 | —16.8216 | .8242 | —16.8229 | + 1.5430 | .5405 | + 1.5418 |
| 58 | —20.6862 | .6824 | —20.6843 | + 0.1187 | .1161 | + 0.1174 |
| 60 | —12.6792 | .6780 | —12.6786 | — 2.6560 | .6565 | — 2.6562 |
| 61 | —23.6384 | .6346 | —23.6365 | — 8.1412 | .1398 | — 8.1405 |
| 62 | —20.0185 | .0185 | —20.0185 | — 6.9216 | .9202 | — 6.9209 |
| 63 | —48.3756 | .3799 | —48.3778 | —21.2167 | .2158 | —21.2162 |
| 64 | —16.4274 | .4340 | —16.4307 | — 7.8912 | .8878 | — 7.8895 |
| 65 | —26.3509 | .3546 | —26.3528 | —38.3480 | .3435 | —38.3458 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE III.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | +51.0658 | .0599 | +51.0628 | —20.6778 | .6808 | —20.6793 |
| 2 | + 9.5818 | .5898 | + 9.5858 | — 4.0546 | .0569 | — 4.0558 |
| 3 | +48.0101 | .0160 | +48.0130 | —14.7761 | .7730 | —14.7746 |
| 4 | +47.0020 | .0083 | +47.0052 | + 1.0397 | .0435 | + 1.0416 |
| 5 | +53.1907 | .1964 | +53.1936 | +12.3321 | .3352 | +12.3336 |
| 6 | +48.4349 | .4345 | +48.4347 | +11.9217 | .9198 | +11.9208 |
| 7 | +33.5885 | .5896 | +33.5890 | + 8.6413 | .6444 | + 8.6428 |
| 8 | +32.6415 | .6342 | +32.5378 | + 9.5983 | .5973 | + 9.5978 |
| 9 | +23.4414 | .4503 | +23.4458 | + 7.4401 | .4399 | + 7.4400 |
| 10 | +45.2597 | .2645 | +45.2621 | +16.2984 | .3038 | +16.3011 |
| 11 | +19.4166 | .4055 | +19.4110 | + 8.5704 | .5667 | + 8.5686 |
| 12 | +37.3367 | .3394 | +37.3380 | +28.0599 | .0574 | +28.0586 |
| 13 | +33.0674 | .0691 | +33.0682 | +25.7809 | .7803 | +25.7806 |
| 14 | +43.0071 | .0052 | +43.0062 | +39.0894 | .0830 | +39.0862 |
| 16 | +39.2033 | .2032 | +39.2032 | +41.8145 | .8135 | +41.8140 |
| 17 | +35.0600 | .0484 | +35.0542 | +39.2671 | .2731 | +39.2701 |
| 18 | +11.0548 | .0613 | +11.0580 | +15.0980 | .0968 | +15.0974 |
| 19 | +21.2808 | .2885 | +21.2846 | +30.9269 | .9234 | +30.9252 |
| 21 | + 8.1541 | .1521 | + 8.1531 | +29.2352 | .2343 | +29.2348 |
| 22 | +10.9249 | .9288 | +10.9268 | +52.4032 | .4129 | +52.4080 |
| 23 | + 0.2139 | .2141 | + 0.2140 | +12.8408 | .8414 | +12.8411 |
| 24 | + 9.0017 | .0075 | + 9.0046 | +55.9556 | .9571 | +55.9564 |
| 25 | — 6.1837 | .1875 | — 6.1856 | +24.9779 | .9756 | +24.9768 |
| 26 | —10.4986 | .4964 | —10.4975 | +19.8676 | .8666 | +19.8671 |
| 28 | —32.5342 | .5300 | —32.5321 | +23.8727 | .8665 | +23.8696 |
| 29 | —34.9903 | .9881 | —34.9892 | +19.1727 | .1713 | +19.1720 |
| 30 | —26.1369 | .1355 | —26.1362 | +13.0860 | .0852 | +13.0856 |
| 31 | —44.2997 | .3009 | —44.3003 | +16.4858 | .4876 | +16.4867 |
| 32 | —36.5603 | .5622 | —36.5612 | + 6.3869 | .3902 | + 6.3886 |
| 33 | —44.3225 | .3271 | —44.3248 | + 7.8843 | .8903 | + 7.8873 |
| 34 | —44.4571 | .4539 | —44.4555 | + 7.8563 | .8518 | + 7.8540 |
| 35 | —40.4034 | .4050 | —40.4042 | + 4.2092 | .2045 | + 4.2068 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE III.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 36 | —17.0294 | .0277 | —17.0286 | + 1.0563 | .0569 | + 1.0566 |
| 38 | —30.0463 | .0419 | —30.0441 | — 4.7414 | .7381 | — 4.7398 |
| 39 | —38.2234 | .2221 | —38.2228 | — 8.7677 | .7677 | — 8.7677 |
| 40 | —41.8617 | .8564 | —41.8590 | —10.7570 | .7576 | —10.7573 |
| 41 | —23.1587 | .1599 | —23.1593 | —10.7451 | .7452 | —10.7452 |
| 42 | —18.5083 | .5010 | —18.5046 | —11.3711 | .3737 | —11.3724 |
| 43 | —28.6290 | .6234 | —28.6262 | —19.0267 | .0223 | —19.0245 |
| 44 | —37.5490 | .5515 | —37.5502 | —31.8343 | .8300 | —31.8322 |
| 45 | —10.1567 | .1558 | —10.1562 | —10.0404 | .0393 | —10.0398 |
| 46 | —12.8296 | .8305 | —12.8300 | —18.2378 | .2363 | —18.2370 |
| 47 | —28.4242 | .4186 | —28.4214 | —47.4485 | .4472 | —47.4478 |
| 48 | — 5.1991 | .1916 | — 5.1954 | — 5.2482 | .2492 | — 5.2487 |
| 49 | — 5.2115 | .2174 | — 5.2144 | — 5.4449 | .4447 | — 5.4448 |
| 50 | —22.9428 | .9385 | —22.9406 | —45.7960 | .7905 | —45.7932 |
| 51 | —10.5335 | .5284 | —10.5310 | —28.6725 | .6739 | —28.6732 |
| 52 | —10.9744 | .9685 | —10.9714 | —43.3558 | .3553 | —43.3556 |
| 53 | — 6.5189 | .5244 | — 6.5216 | —20.2237 | .2259 | —20.2248 |
| 54 | — 7.4963 | .4987 | — 7.4975 | —40.6662 | .6672 | —40.6667 |
| 56 | — 2.8678 | .8724 | — 2.8701 | —26.9716 | .9709 | —26.9712 |
| 58 | — 0.9128 | .9095 | — 0.9112 | —21.8442 | .8440 | — 2.8441 |
| 60 | + 1.8461 | .8504 | + 1.8482 | —13.8325 | .8316 | —13.8320 |
| 61 | + 7.3620 | .3679 | + 7.3650 | —24.7739 | .7753 | —24.7746 |
| 62 | + 6.1341 | .1292 | + 6.1316 | —21.1636 | .1627 | —21.1632 |
| 63 | +20.5026 | .5088 | +20.5057 | —49.4836 | .4818 | —49.4827 |
| 64 | + 7.0903 | .0912 | + 7.0908 | —17.5685 | .5656 | —17.5670 |
| 65 | +37.5742 | .5785 | +37.5764 | —27.3972 | .4015 | —27.3994 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE III.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | +51.0823 | .0681 | +51.0752 | —20.2708 | .2714 | —20.2711 |
| 2 | + 9.5958 | .6004 | + 9.5981 | — 3.6515 | .6499 | — 3.6507 |
| 3 | +48.0306 | .0227 | +48.0266 | —14.3676 | .3654 | —14.3665 |
| 4 | +47.0215 | .0218 | +47.0216 | + 1.4458 | .4516 | + 1.4487 |
| 5 | +53.2089 | .2034 | +53.2062 | +12.7388 | .7419 | +12.7404 |
| 6 | +48.4367 | .4402 | +48.4384 | +12.3301 | .3205 | +12.3253 |
| 7 | +33.5960 | .5934 | +33.5947 | + 9.0459 | .0516 | + 9.0488 |
| 8 | +32.6625 | .6418 | +32.6522 | +10.0008 | .0047 | +10.0028 |
| 9 | +23.4577 | .4477 | +23.4527 | + 7.8417 | .8479 | + 7.8448 |
| 10 | +45.2765 | .2755 | +45.2760 | +16.7019 | .7087 | +16.7053 |
| 11 | +19.4301 | .4200 | +19.4250 | + 8.9789 | .9737 | + 8.9763 |
| 12 | +37.3534 | .3520 | +37.3527 | +28.4715 | .4706 | +28.4710 |
| 13 | +33.0791 | .0825 | +33.0808 | +26.1826 | .1962 | +26.1894 |
| 14 | +43.0201 | .0164 | +43.0182 | +39.4910 | .4905 | +39.4908 |
| 16 | +39.2313 | .2293 | +39.2303 | +42.2137 | .2214 | +42.2176 |
| 17 | +35.0781 | .0697 | +35.0739 | +39.6645 | .6872 | +39.6758 |
| 18 | +11.0678 | .0733 | +11.0706 | +15.4971 | .5013 | +15.4992 |
| 19 | +21.2941 | .2952 | +21.2946 | +31.3325 | .3370 | +31.3348 |
| 21 | + 8.1667 | .1621 | + 8.1644 | +29.6423 | .6380 | +29.6402 |
| 22 | +10.9357 | .9386 | +10.9372 | +52.8103 | .8136 | +52.8120 |
| 23 | + 0.2264 | .2253 | + 0.2258 | +13.2470 | .2483 | +13.2476 |
| 24 | + 9.0190 | .0175 | + 9 0182 | +56.3738 | .3797 | +56.3768 |
| 25 | — 6.1674 | .1720 | — 6.1697 | +25.3889 | .3864 | +25.3876 |
| 26 | —10.4843 | .4859 | —10.4851 | +20.2742 | .2780 | +20.2761 |
| 28 | —32.5245 | .5220 | —32.5232 | +24.2765 | .2709 | +24.2737 |
| 29 | —34.9821 | .9849 | —34.9835 | +19.5785 | .5810 | +19.5798 |
| 30 | —26.1179 | .1182 | —26.1180 | +13.4934 | .4944 | +13.4939 |
| 31 | —44.2997 | .3031 | —44.3014 | +16.8885 | .8960 | +16.8922 |
| 32 | —36.5496 | .5542 | —36.5519 | + 6.7906 | .7965 | + 6.7936 |
| 33 | | | | | | |
| 34 | —44.4461 | .4399 | —44.4430 | + 8.2628 | .2593 | + 8.2610 |
| 35 | —40.3890 | .3907 | —40.3898 | + 4.6133 | .6127 | + 4.6130 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE III.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 36 | —17.0130 | .0141 | —17.0136 | + 1.4598 | .4641 | + 1.4620 |
| 38 | —30.0325 | .0274 | —30.0300 | — 4.3330 | .3318 | — 4.3324 |
| 39 | —38.2048 | .2110 | —38.2079 | — 8.3518 | .3513 | — 8.3516 |
| 40 | —41.8452 | .8372 | —41.8412 | —10.3533 | .3514 | —10.3524 |
| 41 | —23.1413 | .1439 | —23.1426 | —10.3393 | .3390 | —10.3392 |
| 42 | —18.4944 | .4934 | —18.4939 | —10.9637 | .9643 | —10.9640 |
| 43 | —28.6107 | .6104 | —28.6106 | —18.6250 | .6163 | —18.6206 |
| 44 | | | | | | |
| 45 | —10.1459 | .1431 | —10.1445 | — 9.6337 | .6355 | — 9.6346 |
| 46 | —12.8163 | .8135 | —12.8149 | —17.8313 | .8232 | —17.8272 |
| 47 | —28.4075 | .4094 | —28.4084 | —47.0432 | .0555 | —47.0494 |
| 48 | — 5.1797 | .1745 | — 5.1771 | — 4.8416 | .8400 | — 4.8408 |
| 49 | — 5.1998 | .2049 | — 5.2024 | — 5.0390 | .0384 | — 5.0387 |
| 50 | —22.9263 | .9280 | —22.9272 | —45.3801 | .3784 | —45.3792 |
| 51 | —10.5238 | .5182 | —10.5210 | —28.2661 | .2606 | —28.2634 |
| 52 | —10.9606 | .9602 | —10.9604 | —42.9490 | .9480 | —42.9485 |
| 53 | — 6.5044 | .5147 | — 6.5096 | —19.8209 | .8183 | —19.8196 |
| 54 | — 7.4815 | .4894 | — 7.4854 | —40.2547 | .2545 | —40.2546 |
| 56 | — 2.8581 | .8596 | — 2.8588 | —26.5635 | .5627 | —26.5631 |
| 58 | — 0.8942 | .8942 | — 0.8942 | —21.4385 | .4400 | —21.4392 |
| 60 | + 1.8561 | .8594 | + 1.8578 | —13.4240 | .4225 | —13.4232 |
| 61 | + 7.3730 | .3706 | + 7.3718 | —24.3671 | .3651 | —24.3661 |
| 62 | + 6.1481 | .1421 | + 6.1451 | —20.7544 | .7558 | —20.7551 |
| 63 | +20.5170 | .5181 | +20.5176 | —49.0678 | .0706 | —49.0692 |
| 64 | + 7.1020 | .1005 | + 7.1012 | —17.1645 | .1623 | —17.1634 |
| 65 | +37.5892 | .5904 | +37.5898 | —26.9907 | .9940 | —26.9924 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE IV.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | —48.9275 | .9304 | —48.9290 | —24.6663 | .6589 | —24.6626 |
| 2 | — 8.4684 | .4684 | — 8.4684 | — 5.6598 | .6594 | —56.6596 |
| 3 | —42.5129 | .5169 | —42.5149 | —26.4724 | .4715 | —26.4720 |
| 4 | —30.2553 | .2615 | —30.2584 | —36.5236 | .5188 | —36.5212 |
| 5 | —26.2267 | .2262 | —26.2264 | —48.7393 | .7422 | —48.7408 |
| 6 | —23.2733 | .2737 | —23.2735 | —44.9856 | .9842 | —44.9849 |
| 7 | —15.5405 | .5411 | —15.5408 | —31.8829 | .8781 | —31.8805 |
| 8 | —14.2023 | .1965 | —14.1994 | —31.8254 | .8296 | —31.8275 |
| 9 | — 9.5185 | .5148 | — 9.5166 | —23.6276 | .6260 | —23.6268 |
| 10 | —17.9002 | .9000 | —17.9001 | —45.6377 | .6377 | —45.6377 |
| 11 | — 5.9224 | .9280 | — 5.9252 | —21.4774 | .4735 | —21.4754 |
| 12 | — 3.9098 | .9055 | — 3.9076 | —47.8616 | .8594 | —47.8605 |
| 13 | — 2.6538 | .6571 | — 2.6552 | —43.2025 | .2003 | —43.2014 |
| 14 | + 0.2944 | .2949 | + 0.2946 | —59.5130 | .5237 | —59.5184 |
| 15 | + 1.0610 | .0672 | + 1.0641 | —52.8019 | .8066 | —52.8042 |
| 17 | + 5.8492 | .8541 | + 5.8516 | —53.8394 | .8515 | —53.8454 |
| 18 | + 4.5333 | .5309 | + 4.5321 | —19.8166 | .8194 | —19.8180 |
| 19 | + 9.1447 | .1460 | + 9.1454 | —38.0768 | .0772 | —38.0770 |
| 20 | +13.6222 | .6173 | +13.6198 | —33.9137 | .9084 | —33.9110 |
| 21 | +16.8630 | .8629 | +16.8630 | —27.3131 | .3117 | —27.3124 |
| 22 | +31.9146 | .9159 | +31.9152 | —45.1488 | .1550 | —45.1519 |
| 23 | +10.2856 | .2878 | +10.2867 | —10.3259 | .3174 | —10.3216 |
| 24 | +35 8404 | .8365 | +35.8384 | —46.1650 | .1634 | —46.1642 |
| 25 | +23.5123 | .5143 | +23.5133 | —13.9313 | .9365 | —13.9339 |
| 26 | +22.7301 | .7277 | +22.7289 | — 7.2927 | .2888 | — 7.2908 |
| 27 | +36.2335 | .2325 | +36.2330 | — 0.0244 | .0236 | — 0.0240 |
| 28 | +40.6579 | .6657 | +40.6618 | + 6.1116 | .1160 | + 6.1138 |
| 29 | +38.9157 | .9089 | +38.9123 | +11.0894 | .0917 | +11.0906 |
| 30 | +28.4185 | .4239 | +28.4212 | + 8.7695 | .7727 | + 8.7711 |
| 31 | +43.3068 | .3055 | +43.3062 | +19.7340 | .7371 | +19.7356 |
| 32 | +30.6207 | .6205 | +30.6206 | +20.9627 | .9666 | +20.9646 |
| 33 | +37.0172 | .0171 | +37.0172 | +25.6128 | .6143 | +25.6136 |
| 34 | +37.0657 | .0680 | +37.0668 | +25.7424 | .7434 | +25.7429 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE IV.

| Star. | X. Big Images. | | | Y. Big Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 35 | +31.6408 | .6401 | +31.6404 | +25.2624 | .2613 | +25.2618 |
| 36 | +13.3996 | .3933 | +13.3964 | +10.3004 | .3027 | +10.3016 |
| 37 | +11.9648 | .9615 | +11.9632 | +10.5731 | .5747 | +10.5739 |
| 38 | +18.0416 | .0413 | +18.0414 | +23.7814 | .7860 | +23.7837 |
| 39 | +20.6756 | .6799 | +20.6778 | +32.5166 | .5151 | +32.5158 |
| 40 | +21.6936 | .6908 | +21.6922 | +36.5194 | .5236 | +36.5215 |
| 41 | + 8.9509 | .9529 | + 8.9519 | +22.8479 | .8414 | +22.8446 |
| 42 | + 5.3365 | .3349 | + 5.3357 | +19.8783 | .8798 | +19.8790 |
| 43 | + 6.6266 | .6217 | + 6.6242 | +32.4812 | .4841 | +32.4826 |
| 44 | + 3.3449 | .3421 | + 3.3435 | +47.7300 | .7343 | +47.7322 |
| 45 | + 0.5952 | .6022 | + 0.5987 | +12.8595 | .8564 | +12.8580 |
| 46 | — 3.5811 | .5801 | — 3.5806 | +20.3921 | .3908 | +30.3914 |
| 47 | —14.3233 | .3237 | —14.3235 | +51.7107 | .7116 | +51.7112 |
| 48 | + 0.7182 | .7213 | + 0.7198 | + 5.9582 | .9579 | + 5.9566 |
| 49 | + 0.5860 | .5897 | + 0.5878 | + 6.1153 | .1171 | + 6.1162 |
| 50 | —16.8371 | .8407 | —16.8389 | +46.5679 | .5683 | +46.5681 |
| 51 | —12.7767 | .7774 | —12.7770 | +25.8171 | .8199 | +25.8185 |
| 52 | —23.2184 | .2221 | —23.2202 | +36.1597 | .1574 | +36.1586 |
| 53 | — 9.3265 | .3225 | — 9.3245 | +17.1343 | .1377 | +17.1360 |
| 54 | —23.6144 | .6160 | —23.6152 | +31.7852 | .7872 | +31.7862 |
| 55 | — 3.4494 | .4492 | — 3.4493 | + 6.6076 | .6078 | + 6.6077 |
| 56 | —16.7355 | .7339 | —16.7347 | +19.0579 | .0641 | +19.0610 |
| 57 | —10.5240 | .5224 | —10.5232 | +12.5441 | .5452 | +12.5446 |
| 58 | —14.3303 | .3362 | —14.3332 | +14.1304 | .1350 | +14.1327 |
| 59 | —20.1089 | .1063 | —20.1076 | +12.9172 | .9219 | +12.9196 |
| 60 | —10.3477 | .3467 | —10.3472 | + 6.6407 | .6465 | + 6.6436 |
| 61 | —22.1260 | .1281 | —22.1270 | +10.0693 | .0752 | +10.0722 |
| 62 | —18.6324 | .6327 | —18.6326 | + 8.5271 | .5287 | + 8.5279 |
| 63 | —49.1427 | .1466 | —49.1446 | +17.3142 | .3125 | +17.3134 |
| 64 | —16.6681 | .6674 | —16.6678 | + 5.3588 | .3622 | + 5.3605 |
| 65 | —44.6232 | .6234 | —44.6233 | —10.2262 | .2300 | —10.2281 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE IV.

| Star. | X. MiddleImages. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 1 | −48.9305 | .9309 | −48.9307 | −24.2827 | .2791 | −24.2809 |
| 2 | − 8.4688 | .4691 | − 8.4689 | − 5.2794 | .2751 | − 5.2772 |
| 3 | −42.5195 | .5177 | −42.5186 | −26.0848 | .0849 | −26.0848 |
| 4 | −30.2573 | .2579 | −30.2576 | −36.1426 | .1365 | −36.1396 |
| 5 | −26.2306 | .2275 | −26.2290 | −48.3606 | .3576 | −48.3591 |
| 6 | −23.2781 | .2802 | −23.2792 | −44.6043 | .6048 | −44.6046 |
| 7 | −15.5425 | .5478 | −15.5452 | −31.4999 | .4921 | −31.4960 |
| 8 | −14.2058 | .2060 | −14.2059 | −31.4416 | .4406 | −31.4411 |
| 9 | − 9.5195 | .5191 | − 9.5193 | −23.2459 | .2453 | −23.2456 |
| 10 | −17.9000 | .9022 | −17.9011 | −45.2512 | .2491 | −45.2502 |
| 11 | − 5.9252 | .9290 | − 5.9271 | −21.0860 | .0873 | −21.0866 |
| 12 | − 3.9075 | .9081 | − 3.9078 | −47.4717 | .4758 | −47.4738 |
| 13 | − 2.6507 | .6568 | − 2.6538 | −42.8175 | .8109 | −42.8142 |
| 14 | + 0.2939 | .2914 | + 0.2926 | −59.1454 | .1421 | −59.1438 |
| 15 | + 1.0556 | .0575 | + 1.0566 | −52.4211 | .4338 | −52.4274 |
| 17 | + 5.8498 | .8515 | + 5.8506 | −53.4517 | .4569 | −53.4543 |
| 18 | + 4.5284 | .5270 | + 4.5277 | −19.4338 | .4333 | −19.4336 |
| 19 | + 9.1397 | .1433 | + 9.1415 | −37.6966 | .6980 | −37.6973 |
| 20 | +13.6214 | .6200 | +13.6207 | −33.5259 | .5212 | −33 5236 |
| 21 | +16.8622 | .8629 | +16.8626 | −26.9335 | .9332 | −26.9334 |
| 22 | +31.9138 | .9126 | +31.9132 | −44.7740 | .7780 | −44.7760 |
| 23 | +10.2833 | .2840 | +10.2836 | − 9.9424 | .9368 | − 9.9396 |
| 24 | +35.8397 | .8335 | +35.8366 | −45.7798 | .7765 | −45.7782 |
| 25 | +23.5091 | .5123 | +23.5107 | −13.5486 | .5510 | −13.5498 |
| 26 | +22.7285 | .7261 | +22.7273 | − 6.9063 | .9071 | − 6.9067 |
| 27 | +36.2337 | .2282 | +36.2310 | + 0.3554 | .3638 | + 0.3596 |
| 28 | +40.6603 | .6667 | +40.6635 | + 6.4926 | .5017 | + 6.4972 |
| 29 | +38.9152 | .9101 | +38.9126 | +11.4706 | .4731 | +11.4718 |
| 30 | +28.4191 | .4222 | +28.4206 | + 9.1536 | .1576 | + 9.1556 |
| 31 | +43.3073 | .3063 | +43.3068 | +20.1135 | .1123 | +20.1129 |
| 32 | +30.6227 | .6188 | +30.6208 | +21.3492 | .3529 | +21.3510 |
| 33 | — | | | | | |
| 34 | +37.0662 | .0693 | +37.0678 | +26.1218 | .1139 | +26.1178 |

TABLE III.—(*Continued.*) CORRECTED COÖRDINATES. PLATE IV.

| Star. | X. Middle Images. | | | Y. Middle Images. | | |
|---|---|---|---|---|---|---|
| | Direct. | Reversed. | Mean. | Direct. | Reversed. | Mean. |
| 35 | +31.6418 | .6416 | +31.6417 | +25.6439 | .6421 | +25.6430 |
| 36 | +13.3991 | .3939 | +13.3965 | +10.6837 | .6853 | +10.6845 |
| 37 | +11.9700 | .9632 | +11.9666 | +10.9728 | .9685 | +10.9706 |
| 38 | +18.0396 | .0410 | +18.0403 | +24.1685 | .1661 | +24.1673 |
| 39 | +20.6796 | .6825 | +20.6810 | +32.9026 | .9054 | +32.9040 |
| 40 | +21.6951 | .6964 | +21.6958 | +36.9140 | .9206 | +36.9173 |
| 41 | + 8.9506 | .9537 | + 8.9522 | +23.2243 | .2232 | +23.2238 |
| 42 | + 5.3362 | .3352 | + 5.3357 | +20.2603 | .2658 | +20.2630 |
| 43 | + 6.6304 | .6252 | + 6.6278 | +32.8633 | .8671 | +32.8652 |
| 44 | + 3.3522 | .3461 | + 3.3492 | +48.1082 | .1255 | +48.1168 |
| 45 | + 0.5934 | .6012 | + 0.5973 | +13.2394 | .2439 | +13.2416 |
| 46 | — 3.5844 | .5844 | — 3.5844 | +20.7754 | .7769 | +20.7762 |
| 47 | —14.3127 | .3129 | —14.3128 | +52.0979 | .0966 | +52.0772 |
| 48 | + 0.7216 | .7223 | + 0.7220 | + 6.3454 | .3473 | + 6.3464 |
| 49 | + 0.5870 | .5895 | + 0.5882 | + 6.5011 | .5028 | + 6.5020 |
| 50 | —16.8363 | .8385 | —16.8374 | +46.9469 | .9522 | +46.9496 |
| 51 | —12.7772 | .7769 | —12.7770 | +26.2011 | .2035 | +26.2023 |
| 52 | —23.2171 | .2208 | —23.2190 | +36.5401 | .5383 | +36.5392 |
| 53 | — 9.3264 | .3231 | — 9.3248 | +17.5215 | .5235 | +17.5225 |
| 54 | —23.6114 | .6141 | —23.6128 | +32.1703 | .1710 | +32.1706 |
| 55 | — 3.4469 | .4482 | — 3.4476 | + 6.9892 | .9975 | + 6.9934 |
| 56 | —16.7358 | .7332 | —16.7345 | +19.4458 | .4485 | +19.4472 |
| 57 | —10.5243 | .5231 | —10.5237 | +12.9294 | .9260 | +12.9277 |
| 58 | —14.3323 | .3352 | —14.3338 | +14.5137 | .5143 | +14.5140 |
| 59 | —20.1055 | .1008 | —20.1032 | +13.3002 | .3028 | +13.3015 |
| 60 | —10.3510 | .3482 | —10.3496 | + 7.0274 | .0295 | + 7.0284 |
| 61 | —22.1244 | .1245 | —22.1244 | +10.4513 | .4586 | +10.4550 |
| 62 | —18.6321 | .6319 | —18.6320 | + 8.9114 | .9147 | + 8.9130 |
| 63 | —49.1432 | .1463 | —49.1448 | +17.6997 | .7009 | +17.7003 |
| 64 | —16.6681 | .6686 | —16.6684 | + 5.7452 | .7452 | + 5.7452 |
| 65 | —44.6400 | .6379 | —44.6390 | — 9.8408 | .8430 | — 9.8419 |

IV.

## Method of Determining the Right Ascension and Declination of a Star from Its Measured Coördinates.

The principles upon which this method is based have been eluci dated and published by Professor Jacoby in the *Astronomical Journal*, Vol. X, p. 120. The same article is to be found in full in Scheiner's Photographie der Gestirne, pp. 159–166, and it is necessary to give here only the application of the theorem to the case in hand.

The problem may be stated thus:

*Given* a series of circles and angles on a sphere which are projected upon a plane tangent to the sphere, becoming thus straight lines and plane angles; *required* to pass from the measured straight lines and plane angles to the corresponding spherical parts.

The solution naturally falls into two divisions. The first case includes those angles which have their vertices at the point of tangency; the second those which have their vertices elsewhere. The second case is discussed in the paper by Jacoby and is the one that concerns us here. In this treatment it is assumed that the photographic plate is perpendicular to the optical axis of the telescope and is cut by it at the intersection of the two middle reseau lines. This point is the point of tangeucy of the plane and sphere, and is thus the center of projection.

Let $r$ be the focal length of the objective. If $B_p$ (using the notation of the paper referred to) is a plane angle one side of which passes through $o$ the center of projection, and the vertex of which is at a distance $\rho$ from $o$, then the correction necessary to change $B_p$ into $B_s$ the corresponding spherical angle is

$$\eta = \frac{1}{2} \frac{\rho^2}{r^2} \sin B_p \cos B_p,$$

$$B_s = B_p + \eta.$$

If $d$ is the linear distance between the vertex $m$ of $B_p$ and some point $m'$ situated on the side of $B_p$ not passing through $o$, and $\rho'$ is the corresponding distance between $o$ and $m'$; then if $s$ is the corresponding distance on the sphere, expressed in seconds of arc,

$$s = \frac{d}{r} - \frac{1}{2}\frac{d}{r^3}\rho^2 \cos^2 B_p - \frac{1}{2}\frac{d}{r^3}\rho_1{}^2 + \frac{1}{6}\frac{d}{r^3} d^2.$$

In applying these formulas to the case of the polar plates, we shall assume that we know the usual constants for each plate, *i. e.*, the coördinates of the pole, the angle of orientation and the scale value. Accordingly let

$X$, $Y$ be the rectangular coördinates of the star,

$\xi$, $\eta$ be the rectangular coördinates of the point on the plate corresponding to the pole of the heavens,

$A$ be the right ascension of the $X$-axis in the sky,

$\omega$ be the scale value,

$\alpha$, $\pi$ be the star's apparent right ascension and polar distance

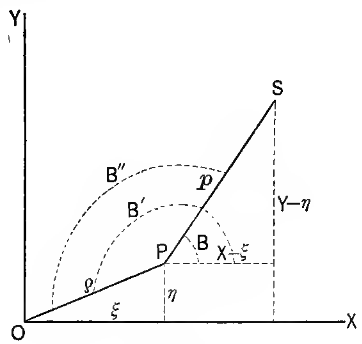


FIG. 2.

From the figure we have the following relations, viz:

$$p \sin B = Y - \eta, \qquad \rho \sin B' = -\eta,$$
$$p \cos B = X - \xi, \qquad \rho \cos B' = -\xi,$$
$$B'' = B' - B.$$

The angle $B$ is the plane angle which we wish to convert into a spherical angle, but neither of its sides passes through the origin, hence the auxiliary angle $B''$ is used, such that $B = B' - B''$.

The correction to be applied to $B$ is

$$\eta = \frac{1}{2}\frac{\rho^2}{r^2} \sin B' \cos B' - \frac{1}{2}\frac{\rho^2}{r^2} \sin B'' \cos B'',$$

$$= \frac{1}{2}\frac{\rho^2}{r^2} \sin B.$$

And thence

$$\alpha = B + \frac{1}{2}\rho^2 \sin B\omega^2 \sin 1'' + A,$$

$$\pi = p\omega - \frac{1}{2}p\rho^2 \cos^2 B''\omega^3 \sin^2 1'' - \frac{1}{2}p(X^2 + Y^2)\omega^3 \sin^2 1''$$

$$+ \frac{1}{6}p^3\omega^3 \sin^2 1''.$$

These formulas hold good if the pole is not more than a degree from the center of the plate. When it is very near the center, as in the 90° plates, certain terms may be omitted. This omission and a slight additional transformation leave the equations in the final form

$$\left.\begin{aligned} \alpha &= B + A, \\ \pi &= p\omega - \frac{1}{3}p^3\omega^3 \sin^2 1'' - p(X\xi + Y\eta)\omega^3 \sin^2 1''. \end{aligned}\right\} \qquad (1)$$

## V.

## Method of Determining the Plate Constants.

The equations (1) can be solved whenever we know the plate constants $\xi$, $\eta$, $A$ and $\omega$, and will furnish the right ascensions and polar distances of the stars on the plates. On the other hand, we can also obtain from them the values of $\xi$, $\eta$, $A$ and $\omega$ if we know the right ascensions and polar distances of at least two stars. If there are more than two known stars on the plate, the resulting equations can be solved by least squares. Since we can obtain very good approximate values of these constants by a direct solution, it will be sufficient to find their corrections by the use of differential formulas derived from equations (1).

If $d\alpha$ and $d\pi$ are changes in $\alpha$ and $\pi$ due to small changes in $\xi$, $\eta$, $A$ and $\omega$, and we put

$$\alpha' = \alpha + d\alpha,$$
$$\pi' = \pi + d\pi,$$

then we have

$$\alpha' = \alpha + \frac{\sin B}{p \sin 1''}d\xi - \frac{\cos B}{p \sin 1''}d\eta + dA,$$

$$\pi' = \pi + pd\omega - \omega \cos Bd\xi - \omega \sin Bd\eta.$$

Of these quantities $a$ and $\pi$ are computed from (1) with the approximate plate constants, $a'$ and $\pi'$ are the apparent places of the standard stars obtained by reduction from some catalogue. Each equation in right ascension should be multiplied by $p\omega \sin 1''$ to give it the same weight as the polar distance equations. If we make this multiplication and adopt the following substitutions

$$n_x = (a - a')\, p\omega \sin 1'',$$
$$n_y = \pi - \pi',$$
$$dA' = \omega \sin 1''\, dA,$$

our equations become

$$\begin{aligned} \omega \sin B d\xi - \omega \cos B d\eta + p dA' + n_x &= 0, \\ -\omega \cos B d\xi - \omega \sin B d\eta + p d\omega + n_y &= 0. \end{aligned} \qquad (2)$$

Each star furnishes two such equations and a least squares solution of the equations furnished by all the standard stars on the plate will give the most probable values of $d\xi$, $d\eta$, $dA'$ and $d\omega$.*

On account of the symmetry of the coefficients in these last two equations the least squares solution can be simplified. Let

$$-\omega \sin B = r, \qquad -\omega \cos B = s,$$

then the equations (2) become

$$-r d\xi + s d\eta + p dA' + 0 d\omega + n_x = 0,$$
$$+s d\xi + r d\eta + 0 dA' + p d\omega + n_y = 0.$$

If $a$, $b$, $c$, $d$, $n$, and $s$, have the signification usually adopted in the Gaussian method of elimination, and the terms indicated in the square bracket are formed in the customary way, we have the following relations,

$$[bb] = [aa], \quad [ab] = 0, \quad [ac] = -[bd], \quad [ad] = [bc],$$
$$[cc] = [dd], \quad [cd] = 0.$$

We can also make use of the ordinary check formulas,

$$[aa] + [ac] + [ad] + [an] = [as],$$
$$[aa] + [ad] - [ac] + [bn] = [bs],$$
$$[ac] + [ad] + [cc] + [cn] = [cs],$$
$$[ad] - [ac] + [cc] + [dn] = [ds],$$

* These formulas appear in a paper written by Professor Jacoby entitled "Photographic Researches near the Pole of the Heavens," and published in the Bulletin of the Imperial Academy of Sciences of St. Petersburg, V Series, Volume IX, No. 1 (June, 1898).

If we place

$$F = [aa] \cdot [cc] - [ac] \cdot [ac] - [ad] \cdot [ad],$$
$$C = [ac] \cdot [cn] + [ad] \cdot [dn] - [cc] \cdot [an],$$
$$D = [ad] \cdot [cn] - [ac] \cdot [dn] - [cc] \cdot [bn],$$
$$G = [ac] \cdot [an] + [ad] \cdot [bn] - [aa] \cdot [cn],$$
$$H = [ad] \cdot [an] - [ac] \cdot [bn] - [aa] \cdot [dn],$$

then

$$\left.\begin{array}{l} d\xi = \frac{C}{F}, \quad d\eta = \frac{D}{F}, \quad dA' = \frac{G}{F}, \quad d\omega = \frac{H}{F}, \\ \text{weight of } d\xi = \text{weight of } d\eta = \frac{F}{[cc]}, \\ \text{weight of } dA' = \text{weight of } d\omega = \frac{F}{[aa]}. \end{array}\right\} \quad (3)$$

Another method of solving these equations has been published by Professor Jacoby,* the study of which suggested to me the above process. The present formulas, however, seem more suitable for this work, particularly since they give a desirable check to the computation early in its course. Their symmetry and simplicity are at once noticeable.

Before giving the results of the several least squares solutions, it will be necessary to explain somewhat in detail the method of computing $a'$ and $\pi'$, the apparent places of the standard stars.

## VI.

## Method of Determining the Apparent Places of the Standard Stars.

The plate constants $\xi$ and $\eta$ are the rectangular coördinates of that point on the plate which is the projection of the pole of the heavens, but since the celestial pole is a moving point its projection is not fixed, and we are at liberty within certain limits to make a choice of the epoch which fixes the exact position of the pole. $\xi$ and $\eta$ are determined through the solution of equations (2) by comparing the apparent places of the stars in the sky with their positions obtained from the measured coördinates, hence the manner of computing their apparent places must be consistent

* Monthly Notices R. A. S., May, 1896.

with the choice of pole. If, pursuing the usual process, we correct the star places taken from some standard catalogue for precession, nutation, aberration and refraction, $\xi$ and $\eta$ will be the coördinates of the pole at the instant of observation, and $A$ will be the right ascension of the $X$-axis referred to the equinox of date. The places of the unknown stars obtained by the solution of equations (1) will be the apparent places at the instant of observation, and must be corrected for refraction and reduced to the beginning of the year, after which a further application of precession is necessary to bring them to any desired epoch.

It is possible to abbreviate quite considerably the labor of this long computation by selecting the position of the pole for some other instant than that of observation, for example, the epoch of the catalogue from which the standard stars are taken. Precession and nutation do not affect the positions of the stars relative to one another, but merely change the planes of reference. Hence if we apply to the right ascensions and polar distances of the standard stars, proper motion, aberration and refraction, the constants $\xi$, $\eta$ and $A$ will give the position on the plate of the pole of the epoch of the catalogue. The scale value, $\omega$, depends upon the mutual distances of the stars and would not be altered by this method of procedure. The right ascensions and polar distances of the unknown stars computed from equations (1) will be their apparent places, but referred to the pole and equinox of the epoch of the catalogue.

We can also make a simplification in the computation of refraction, which would ordinarily be an involved process with stars of such high declination. The pole of the selected epoch may be treated as if it were a luminous point and affected by refraction, in which case the right ascensions and polar distances of all the stars will be affected by differential refraction only. This assumption is allowable, because the point on the plate, of which $\xi$ and $\eta$ are the coördinates, in an imaginary point, and the point in the sky of which it is the projection can be connected with the real pole by means of a simple differential relation.

However, precession affects certain quantities which appear in the formulas for aberration and refraction, and must therefore be considered in finding the place of the refracted pole. The aberration formulas involve the longitude of the sun and the obliquity of the ecliptic; the refraction formulas, the sidereal time, which is the angle formed at the pole by the equinoctial colure and the meridian, and the zenith distance of the pole, all of which quantities are changed by precession. The modifications introduced for each of these changes will be given with the statement of the formulas employed.

## 1. Aberration.

The formulas used are due to Dr. G. W. Hill and are found in the Star Tables of the American Ephemeris, page xxii, where they are numbered (30).

$$\left.\begin{aligned} \alpha - \alpha_1 = & -20''.4451 \operatorname{cosec} \pi_1 [\sin \alpha_1 \sin \odot + \cos \alpha_1 \cos \varepsilon \cos \odot] \\ & - 0''.0009329 \operatorname{cosec}^2 \pi_1 \sin 2\alpha_1 \cos 2 \odot \\ & + 0''.0009295 \operatorname{cosec}^2 \pi_1 \cos 2\alpha_1 \sin 2 \odot \\ & + 0''.311 \cos \phi \cos (\theta - \alpha_1) \operatorname{cosec} \pi_1 , \\ \pi - \pi_1 = & + 20''.4451 \cos \pi_1 \cos \alpha_1 \sin \odot \\ & - 20''.4451 \cos \odot [\cos \pi_1 \sin \alpha_1 \cos \varepsilon - \sin \pi_1 \sin \varepsilon] \\ & + 0''.0004648 \cot \pi_1 \sin 2\alpha_1 \sin 2 \odot \\ & - [0''.0000402 - 0''.00004665 \cos 2\alpha_1] \cot \pi_1 \cos 2 \odot \\ & - 0''.311 \cos \varphi \sin (\theta - \alpha_1) \cos \pi_1 . \end{aligned}\right\} \quad (4)$$

In these equations $\alpha_1$ and $\pi_1$ are the places unaffected by aberration; $\alpha$ and $\pi$ are affected by aberration. In order to use the right ascension and polar distance directly from the catalogue, it will be necessary to apply to the longitude of the sun for date, precession in longitude from the epoch of the catalogue to date of exposure, and to use for $\varepsilon$ the mean obliquity of the ecliptic for the date of catalogue. We shall then have the places of the stars, affected by aberration but referred to the pole and equinox of the date of the catalogue. The last term in each equation gives the effect of diurnal aberration.

## 2. Proper Motion.

The effect of proper motion can be computed by means of the following equations in which $\alpha_2$ and $\pi_2$ are the coördinates of the star unaffected by proper motion. They are also taken from the Star Tables of the American Ephemeris and are numbered (10).

$$\left.\begin{aligned} a - a_2 &= \mu t + \mu\mu' \cot \pi_2 t^2 - \frac{1}{3}[\mu^3 \cos^2 \pi_2 - \mu\mu'^2(1 + 3\cot^2 \pi_2)]t^3, \\ \pi - \pi_2 &= -\mu' t + \frac{1}{4}\mu^2 \sin^2 \pi_2 t^2 + \frac{1}{6}\mu^2\mu'(1 + 2\cos^2 \pi_2)t^3. \end{aligned}\right\} \quad (5)$$

## 3. Refraction.

Very precise formulas for differential refraction have been derived by Jacoby and used by him in his paper on the Pleiades.* At his suggestion the differential method was also employed in this investigation, though several modifications were necessary because of its application in the vicinity of the pole. These modifications were developed by me. Jacoby's formulas give the effect of differential refraction on position angle and distance, hence if we suppose the imaginary refracted pole to take the place of the central star, they will apply equally well to the present case, for the position angle becomes the hour angle, and the measured distance becomes the polar distance. The zenith distance of the pole at Helsingfors is 30°, therefore terms of higher order than the first were omitted. With our notation, Jacoby's group (c) becomes

$$\left.\begin{aligned} a - a' &= + k \cot^2 \phi \sin(\theta - a') \cos(\theta - a'), \\ \pi - \pi' &= + k\pi [\cot^2 \phi \cos^2(\theta - a') + 1] \sin 1''. \end{aligned}\right\} \quad (6)$$

If we hold to the original signification of these formulas, $a'$ and $\pi'$ are the apparent right ascension and polar distance of a star, $\theta$ is the sidereal time of exposure, $z$ is the zenith distance of the pole, and $\theta$ and $a'$ have their vertices at the same point. But if the quantities $a'$ and $\pi'$ are referred to the pole and equinox of the date of the catalogue, in which case $a'$ has its vertex at this pole, it will be necessary to find the angle at this point corresponding to the angle $\theta$, *i. e.*, an angle such that one side passes through the equinox of the adopted epoch, and the other through the zenith at the instant of exposure. By means of differential formulas we can easily pass from one angle to the other in the manner described below. We shall find first the position of the

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* Annals, New York Academy of Science, Vol. VI, p. 253.

pole of the adopted epoch referred to the pole of date, and then pass to its refracted position.

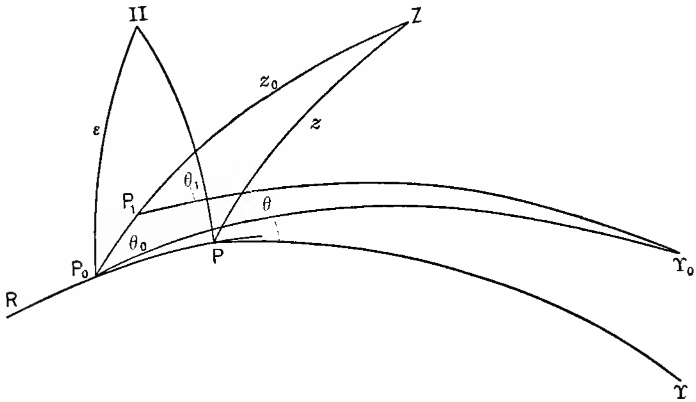


FIG. 3.

Let

$$P_0 = \text{pole at epoch of catalogue,}$$
$$P = \text{pole at date of exposure,}$$
$$P_1 = \text{pole at epoch, when refracted,}$$
$$\Pi = \text{pole of ecliptic,}$$
$$Z = \text{zenith.}$$
$$P_0\Upsilon_0 = \text{equinoctial colure for epoch,}$$
$$P\Upsilon = \text{equinoctial colure for date,}$$
$$ZP\Upsilon = \theta = \text{sidereal time of exposure,}$$
$$ZP_0\Upsilon_0 = \theta_0,\ ZP_1\Upsilon_0 = \theta_1,$$
$$\Upsilon_0 P_0 R = A_0,\ \Upsilon P P_0 = A,$$
$$PZ = z,\ P_0Z = z_0,\ P_1Z = z_1,$$
$$\Pi P_0 = \varepsilon,\ PP_0 = \pi.$$

$A$ is the right ascension of the pole of epoch of catalogue referred to the pole of the date of exposure. It is a little greater than 180°.

$A_0$ is always less than 180°, and $180 + A_0$ is right ascension of $P$ reckoned from $P_0$.

From the figure we have the following relations :

$$\theta = A - ZPR, \qquad \theta_0 = A_0 - ZP_0R,$$

$$\theta_0 = \theta + (A_0 - A) - (ZP_0R - ZPR). \qquad (6)$$

Employing a series developed by Jordan,* we have from the triangle $\Pi PP_0$

$$(270 - A_0) - (270 - A) = \pi \sin (270 - A) \cot \varepsilon,$$

or

$$A_0 - A = \pi \cos A \cot \varepsilon,$$

and from the triangle $ZPP_0$

$$ZPR - ZP_0R = \pi \sin (A - \theta) \cot z.$$

Hence

$$\theta_0 - \theta = \pi[\cos A \cot \varepsilon + \sin (\theta - A) \cot z]. \tag{7}$$

We have also

$$z_0 - z = - \pi \cos (\theta - A). \tag{8}$$

We can use the same differential formula for finding the angle $\theta_1 = ZP_1 \Upsilon_0$;

$$\theta_1 - \theta_0 = \pi \sin \theta_0 \cot P_0 \Upsilon_0 = 0,$$

since

$$P_0 \Upsilon_0 = 90^\circ.$$

To find $z_1$ we have only to find the mean refraction for the zenith distance $z_0$ and apply it directly to $z_0$. Our formulas then read

$$\left.\begin{aligned} a - a' &= + k \sin (\theta_0 + a') \cos (\theta_0 - a') \tan^2 z_1, \\ \pi - \pi' &= + k\pi' [1 + \cos^2 (\theta_0 - a') \tan^2 z_1] \sin 1''. \end{aligned}\right\} \tag{9}$$

For diurnal aberration also we should use $z_1$ and $\theta_0$.

It has been tacitly assumed that we already possess values of $A$ and $\pi$ which are the right ascension and polar distance of the pole of date of exposure. They may be obtained in several ways. We may use Chauvenet's Astronomy I (664), or the method of Fabritius, or more briefly the ordinary precession constant for $t_0$, the epoch of the catalogue, as follows:

$$\left.\begin{aligned} A &= 180^\circ + \frac{1}{2} m (t - t_0), \\ \pi &= n (t - t_0). \end{aligned}\right\} \tag{10}$$

It is desirable to be more precise with nutation since its effect is much greater. This can be accomplished by using the ordinary Besselian constants $A$ and $B$ of the Berliner Jahrbuch, and improving the results by the use of the Fabritius formulas.† The

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* Handbuch der Vermessungskunde, dritte Auflage, 1890, Vol. III, p. 313.

† Astr. Nach., 2072 and 2063; Oppolzer, Bahnbestimmung, Vol. I, p. 258.

aberration terms $Cc$ and $Dd$ are of course omitted. These improved values of $A$ and $\pi$ should be used in equations (7) and (8).

They should not be confounded with the $A$ and $\pi$ of equation (1) and for convenience might be called $A'''$ and $\pi'''$.

$$\left.\begin{aligned}\theta_0-\theta&=\pi'''\,[\cos A'''\cot\varepsilon+\sin(\theta-A''')\cot z],\\ z_0-z&=-\pi'''\cos(\theta-A''').\end{aligned}\right\}\qquad(11)$$

If we determine the right ascensions and polar distances of the known stars in this manner and substitute them in equations (2) for the purpose of finding the plate constants, the values of $\xi$ and $\eta$ thus derived will be the coördinates of the pole of the epoch of the catalogue, considered to be refracted. $A$ will be the right ascension of the $X$-axis reckoned from the equinox of the catalogue, and $\omega$ should remain unaffected by the method adopted.

Conversely, the right ascensions and polar distances of the unknown stars obtained by substituting these values of the constants in (1) should be freed from differential refraction, and aberration and they will then be referred to the pole and equinox of the catalogue.

## VII.

## Determination of Plate Constants.

The standard stars were taken from Elkin's "Heliometer Triangulation of Stars in the Vicinity of the North Pole."* The epoch of this catalogue is 1888.0. The Helsingfors plates are two degrees square, and it was therefore decided to include in the present publication only those stars which lie within one degree of the pole, and consequently on all four plates. Unfortunately only two of Elkin's stars lie in this region, hence it was necessary to utilize a few other stars which were farther from the center and lay in the corners of the plates. The following table includes all of Elkin's stars which were available.† There were a few others appearing on some plates which lay farther from the center than $\omega$, but they were not included.

* Transactions of Yale Observatory, Vol. I, part III.

† Elkin, *loc. cit.*, p. 179.

| Star. | δ | ι | ν | ρ | φ | ω |
|---|---|---|---|---|---|---|
| α | 41° 26′ 32″ | 116° 8′ 33″ | 193° 33′ 33″ | 246° 46′ 54″ | 293° 55′ 50″ | 334° 27′ 22″ |
| π | 1297″.25 | 3728″.37 | 3718″.88 | 2685″.92 | 3735″.81 | 3953″.79 |

Table IV contains the rectangular coördinates of these stars as they appear on the four plates. Each star was measured twice and the mean taken for the final coördinate.

TABLE IV.—COÖRDINATES OF STANDARD STARS.

| | δ | ι | ν | ρ | φ | ω |
|---|---|---|---|---|---|---|
| I B *X* | +18.7952 | +40.4762 | —37.2292 | —45.5262 | —43.6778 | |
| *Y* | — 7.9222 | +48.3180 | +51.8052 | + 2.1541 | —43.6150 | |
| I M *X* | +18.7940 | +40.4924 | —37.2241 | —45.5230 | —43.6883 | |
| *Y* | — 7.5812 | +48.6676 | +52.1562 | + 2.5078 | —43.2487 | |
| II B *X* | + 7.8014 | | | —30.4085 | —61.5584 | —48.4102 |
| *Y* | —19.0512 | | | +33.6557 | + 0.0757 | —42.5585 |
| II M *X* | + 7.7881 | | | —30.4278 | —61.5679 | —48.4162 |
| *Y* | —18.6642 | | | +34.0496 | + 0.4628 | —42.1628 |
| III B *X* | +17.7945 | | —64.0614 | —34.8150 | +41.4649 | |
| *Y* | +6.6870 | | + 9.2847 | —31.6872 | —49.4535 | |
| III M *X* | +17.8049 | | —64.0529 | —34.8028 | +41.4721 | |
| *Y* | + 7.0924 | | + 9.6910 | —31.2800 | —49.0484 | |
| IV B *X* | — 6.2153 | +50.7758 | +51.5005 | + 1.5743 | —44.0910 | |
| *Y* | —18.9971 | —38.6665 | +39.1071 | +45.6420 | +42.1716 | |
| IV M *X* | — 6.2164 | +50.7760 | +51.5053 | + 1.5742 | —44.0907 | |
| *Y* | —18.6091 | —38.2861 | +39.4929 | +46.0252 | +42.5576 | |

I have also collected in tabular form the various quantities needed in the solution of equations (4), (9) and (11).

### Table V.

| | Plate I. | | Plate II. | | Plate III. | | Plate IV. | |
|---|---|---|---|---|---|---|---|---|
| | Big Im. | Middle Im. | Big Im. | Middle Im. | Big Im. | Middle Im. | Big Im. | Middle Im. |
| 1895 | Sept. 16. no. 3 | | Sept. 16. no. 6. | | Sept. 18. no. 2. | | Sept. 21. no. 14. | |
| $\theta$ | $22^h11^m2^s$ | $23^h16^m34^s$ | $0^h24^m59^s$ | $0^h30^m5^s$ | $20^h39^m9^s$ | $20^h44^m13^s$ | $3^h15^m34^s$ | $3^h20^m42^s$ |
| $\odot$ | 173°38′45″ | 173°38′59″ | 173°41′45″ | 173°41′58″ | 175°29′31″ | 173°29′43″ | 178°41′17″ | 178°41′28″ |
| $A'''$ | 183°26′15″ | | 183°26′18″ | | 183°27′15″ | | 183°25′47″ | |
| $\pi'''$ | 156″.79 | | 156″.79 | | 156″.84 | | 156″.90 | |
| $\theta_0-\theta$ | −4′47″ | −4″.53 | −6′14° | −6′20″ | −2′21″ | −2′24″ | −9′16″ | −9′20″ |
| $\theta_0$ | 347°40′43″ | 349°3′37″ | 6°8′31″ | 7°24′55″ | 309°44′54″ | 311°0′51″ | 48°44′14″ | 50°1′10 |
| $z_0-z$ | +2′31″ | +2′32″ | +2′37″ | +2′36″ | +1′33″ | +1′36″ | +1′50″ | +1′47″ |
| $z_1-z$ | +33″ | +33″ | +33″ | +33″ | +33″ | +33″ | +33″ | +33″ |
| $z_1$ | 29°53′21″ | 29°53′22″ | 29°53′27″ | 29°53′26″ | 29°52′23″ | 29°52′26″ | 29°52′40″ | 29°52′37″ |
| cot $\phi_1$ | 9.75949 | 9.75950 | 9.75953 | 9.75952 | 9.75921 | 9.75923 | 9.75929 | 9.75928 |
| log $k$ | 1.76890 | | 1.76949 | | 1.76439 | | 1.78447 | |
| $a$ center | 336° | | 21° | | 291° | | 66° | |
| | $\phi$ 60°9′17″ | | $\varepsilon$ 23°27′14″ | | $\delta$ center 90° | | | |

## Remarks.

1. The Helsingfors sidereal time corresponds to the middle of the time of exposure; see Table I, page 3.

2. The longitude of the sun was taken from the Berliner Jahrbuch for 1895. It is there referred to the mean equinox of the beginning of the year 1895 and was therefore decreased by the precession in longitude for seven years.

3. The obliquity of the ecliptic is the mean obliquity for 1888.0. It is taken from the Jahrbuch for 1888.

4. The effect of precession on the pole of 1888.0 was computed by Chauvenet I. (664). The resulting right ascension and polar distance of pole of 1888 referred to the pole of date are

$$A = 180°2'43''.34$$
$$\pi = 140''.37$$

The values of $A'''$ and $\pi'''$ given in the table are obtained by the ordinary "reduction to date," using the Besselian numbers $A$ and $B$ taken from the Jahrbuch. They were further improved by the use of Fabritius's method.

5. The value of log $k$ was taken from the Pulkowa refraction tables, in accordance with the data given in Table I. It was increased by $\frac{1}{65}$ of itself to allow for the difference between visual and photographic refractions.

I shall now give a rather full outline of the process of reducing one plate, selecting for this purpose plate I, middle images. Annual aberration was computed according to equations (4); diurnal aberration according to the last term of (4) using $\theta_0$ and $90 - z_1$, for $\theta$ and $\varphi$; differential refraction according to (9). The only standard star whose proper motion has been determined is "$\varphi$" or $\lambda$ *Ursæ Minoris.* Its proper motion was taken from the Berliner Jahrbuch, and applied according to the first term of (5).

| Star. | δ | ι | ν | ρ | φ |
|---|---|---|---|---|---|
| | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ | ° ′ ″ |
| α 1888.0 | 41 26 23 | 116 8 33 | 193 33 33 | 246 46 54 | 293 55 50 |
| Annual aberr. | + 32 40 | — 9 24 | — 16 18 | — 6 48 | + 8 54 |
| Prop. Motion. | — | — | — | — | — 6 |
| Diurn. aberr. | + 15 | — 5 | — 8 | — 3 | + 5 |
| Differ. refrac. | + 9 | — 9 | + 7 | + 4 | — 9 |
| α apparent. | 41 59 36 | 115 58 55 | 193 17 14 | 246 40 7 | 294 4 34 |
| | ″ | ″ | ″ | ″ | ″ |
| π 1888.0 | 1297.25 | 3728.37 | 3718.88 | 2685.92 | 3735.81 |
| Annual aberr. | + 13.97 | + 15.58 | — 6.70 | — 18.16 | — 16.28 |
| Prop. Motion. | — | — | — | — | + 0.05 |
| Diurn. aberr. | + 0.12 | + 0.12 | — 0.06 | — 0.15 | — 0.13 |
| Differ. refrac. | — 0.42 | — 1.19 | — 1.35 | — 0.77 | — 1.17 |
| π apparent. | 1310.92 | 3742.88 | 3710.77 | 2666.84 | 3718.28 |

The plate constants are to be determined from equations (2). Approximate values were found quite easily. $A$, the orientation angle of the $X$-axis was placed equal to 66°, *i. e.*, the right ascension of the center of the plate taken from Table V, plus 90°. $\omega$ was assumed to be equal to 60″; $\xi$ and $\eta$, were obtained from the solution of the following simple relations:

$$\omega\eta = \pi''' \sin\,(A''' - A),$$
$$\omega\xi = \pi''' \cos\,(A''' - A).$$

A preliminary solution of (2) with equations derived from two stars gave approximate values of all the constants. This approximation was so close that the further corrections did not exceed $0^{mm}.01$ for $\xi$ and $\eta$, and $0''.01$ for $A$ and $\omega$, and the absolute terms with a few exceptions were less than 1″.

The following specimen equation of condition

$$-26d\xi - 54d\eta + 22dA' + 0''.37 = 0$$

shows that the coefficients and absolute terms are not of the same order of magnitude. In order to simplify the subsequent numerical work each coefficient was divided by 100, and each unknown quantity multiplied by 100, and the following substitutions made:

$$x = 100d\xi, \quad y = 100d\eta, \quad z = 100dA', \quad t = 100d\omega.$$

We then have the following equations:

| | | | | |
|---|---|---|---|---|
| $-.26x$ | $-.54y$ | $+.22z$ | $+.00t$ | $+.37'' = 0$ |
| $+.45$ | $-.40$ | $+.63$ | $+.00$ | $-.11 = 0$ |
| $+.49$ | $+.35$ | $+.62$ | $+.00$ | $+.07 = 0$ |
| $+.01$ | $+.60$ | $+.45$ | $+.00$ | $+.01 = 0$ |
| $-.43$ | $+.41$ | $+.62$ | $+.00$ | $-.29 = 0$ |
| $-.54$ | $+.26$ | $+.00$ | $+.22$ | $+.19 = 0$ |
| $-.40$ | $-.45$ | $+.00$ | $+.63$ | $+.11 = 0$ |
| $+.35$ | $-.49$ | $+.00$ | $+.62$ | $+.00 = 0$ |
| $+.60$ | $-.01$ | $+.00$ | $+.45$ | $-.43 = 0$ |
| $+.41$ | $+.43$ | $+.00$ | $+.62$ | $-.84 = 0$ |

Solving these equations according to equations (3), we have the following values of the unknown quantities and their weights.

$$d\xi = +0^{mm}.0037, \quad \text{weight } 16500,$$
$$d\eta = +0^{mm}.0041, \quad \text{weight } 16500,$$
$$dA' = -0''.0009, \quad \text{weight } 13000,$$
$$dA = -3'',$$
$$d\omega = +0''.0042, \quad \text{weight } 13000,$$

The final coördinates with their probable errors are

$$\xi = -0^{mm}.9734 \pm 0''.0014, \quad \pm 0^{mm}.0002,$$
$$\eta = +1^{mm}.8549 \pm 0''.0014, \quad \pm 0^{mm}.0002,$$
$$A = 67°30'47'' \pm 6'',$$
$$\omega = 59''.8585 \pm 0''.0016.$$

The probable error $r_1$, of one determination of polar distance is $\pm 0''.18$, and of one determination of right ascension is $\pm 0.''18$ cosec $\pi$.

Similar least squares reductions of the remaining plates give rise to the following values of the plate constants.

TABLE VI.—PLATE CONSTANTS.

| Plate. | $\xi$ | $\eta$ | Probable error of $\xi$ or $\eta$. | $A$ | Prob. err. $A$ | $\omega$ | Probable error of $\omega$ | $r_1$ |
|---|---|---|---|---|---|---|---|---|
| | mm | mm | '' | ° ' '' | '' | '' | '' | '' |
| I B | −0.9745 | +1.5051 | ±0.0016 | 67 30 10 | ±6 | 59.8594 | ±0.0018 | ±0.20 |
| M | −0.9734 | +1.8549 | ±0.0014 | 67 30 47 | ±6 | 59.8585 | ±0.0016 | ±0.18 |
| II B | +0.5491 | +1.6113 | ±0.0012 | 112 39 30 | ±6 | 59.8554 | ±0.0017 | ±0.11 |
| M | +0.5353 | +2.0038 | ±0.0018 | 112 39 18 | ±7 | 59.8579 | ±0.0021 | ±0.16 |
| III B | −2.8448 | −0.6334 | ±0.0016 | 22 29 16 | ±6 | 59.8527 | ±0.0019 | ±0.19 |
| M | −2.8349 | −0.2276 | ±0.0013 | 22 29 18 | ±6 | 59.8530 | ±0.0016 | ±0.15 |
| IV B | +2.4822 | +1.0837 | ±0.0017 | 155 28 17 | ±6 | 59.8568 | ±0.0019 | ±0.22 |
| M | +2.4825 | +1.4682 | ±0.0017 | 159 28 24 | ±6 | 59.8554 | ±0.0019 | ±0.22 |

## VIII.

## Results of the Measures.

The plate constants given in the preceding section were used in determining the right ascensions and polar distances of the unknown stars.

The process employed was the reverse of that followed on page 40. One additional step, however, was necessary in computing the aberration which according to equations (4) involves in the second member $\alpha_1$ and $\pi_1$ which are unaffected by aberration.

A first determination of aberration was made by using the first two terms of (4) and the resulting $\Delta\alpha$ and $\Delta\pi$ applied to the right ascensions and polar distances which had been already freed from the effects of differential refraction and diurnal aberration. This gives approximately the true places for 1888.0, and these new values of $\alpha$ and $\pi$ were substituted in the complete equations (4). This additional step was taken but once, and the approximate places served as a basis for computing the aberration for all four plates.

The reductions of the big and middle images have been carried through to the end independently, with the exception of the determination of annual aberration. The difference in the times of exposure corresponding to the big and middle images had no effect on annual aberration, hence but one determination was made for each plate. The mean of the values deduced from the big and middle images on each plate was taken as the mean right ascension and polar distance for that plate, and the final right ascension and polar distance is the mean of the four results thus obtained. The residuals are the differences between the final mean and the mean for each plate.

In the table of mean results immediately following, the figures in the second column refer to the number of the plate and the image " big " or " middle " which is reduced. The columns headed " $\alpha$ " and " $\pi$ " give the right ascension and polar distance for each image measured. The columns headed $\frac{1}{2}(B+M)$ give the mean of the $\alpha$ or $\pi$ for the big and middle images on any one plate. The " mean " is the mean of these last quantities and not of the quantities in the columns " $\alpha$ " or " $\pi$ ". The residuals are the differences between this mean and the mean for each plate.

The probable error of the mean place was derived from the probable errors, $r_1$, of the single determinations of right ascension and polar distance for each plate. The means from the four plates were treated as four independent observations, and the ordinary formula for propagation of error was used,

$$r_0 = \pm \frac{\sqrt{r_1^2 + r_2^2 \ldots + r_n^2}}{n}. \tag{12}$$

This will give the same probable error for stars which are found on all four plates, viz:

for $\alpha$, $r_0 = \pm 0''.064 \operatorname{cosec} \pi$,
for $\pi$, $r_0 = \pm 0''.064$.

The probable errors of the stars which do not appear on all four plates were obtained by substituting in equation (12) only the probable errors of the plates on which they were found. The probable errors are attached to the means in the table of mean results.

It will be noticed that in taking the final mean, equal weights were given to the means from the several plates, although this is not in accordance with a rigorous interpretation of the principles of least squares, whereby these means should be given weights inversely proportional to the squares of the probable errors.

This method was adopted because the conditions involved did not warrant in my opinion a strict following of the least squares method. In fact if I had proceeded on the basis that the probable errors were equal for the different plates, and had found an average probable error by taking the mean of the eight probable errors for the different images, and if I had used this average probable error in the formula

$$r_0 = \frac{\sqrt{nr^2}}{n},$$

exactly the same result, $r_0 = \pm 0''.064$ would have been obtained.

For an explanation of the asterisk in column 1 Table VII see p. 68.

## Table VII.—Mean Results.

| Star. | Plate. | α | ½(B+M) | Δα sin π | π | ½(B+M) | Δπ |
|---|---|---|---|---|---|---|---|
| *1 | I B | 1° 45′ 50″ | 46′ 2″ | —.05″ | 3441.53″ | 1.64″ | +.22″ |
| | I M | 46 15 | | | 1.74 | | |
| | II B | 45 40 | 45 50 | —.26 | 0.85 | 1.11 | —.31 |
| | II M | 45 59 | | | 1.37 | | |
| | III B | 46 32 | 46 38 | +.56 | 1.05 | 1.10 | —.32 |
| | III M | 46 45 | | | 1.16 | | |
| | IV B | 45 44 | 45 50 | —.26 | 1.78 | 1.82 | +.40 |
| | IV M | 45 56 | | | 1.87 | | |
| | | Means 1° 46′ 5″ ±4″ | | | | 3441.42″ ±.06″ | |
| *2 | I B | 5° 41′ 0″ | 41′ 26″ | —.24″ | 768.30″ | 8.35″ | —.02″ |
| | I M | 41 52 | | | 8.40 | | |
| | II B | 42 33 | 42 34 | +.03 | 8.38 | 8.44 | +.07 |
| | II M | 42 34 | | | 8.50 | | |
| | III B | 43 1 | 43 1 | +.14 | 8 50 | 8.58 | +.21 |
| | III M | 43 1 | | | 8.65 | | |
| | IV B | 42 28 | 42 43 | +.07 | 8.17 | 8.12 | —.25 |
| | IV M | 42 58 | | | 8.07 | | |
| | | Means 5° 42′ 28″ ±16″ | | | | 768.37″ ±.06″ | |
| *3 | I B | 6° 36′ 14″ | 36′ 24″ | —.27″ | 3157.14″ | 7.07″ | +.21″ |
| | I M | 36 34 | | | 7.00 | | |
| | II B | 36 24 | 36 38 | —.06 | 6.79 | 6.96 | +.10 |
| | II M | 36 52 | | | 7.14 | | |
| | III B | 36 43 | 36 46 | +.06 | 6.49 | 6.58 | —.28 |
| | III M | 36 50 | | | 6.67 | | |
| | IV B | 37 6 | 37 2 | +.30 | 6.81 | 6.84 | —.02 |
| | IV M | 36 57 | | | 6.86 | | |
| | | Means 6° 36′ 42″ ±4″ | | | | 3156.86″ ±.06″ | |
| *4 | I B | 24° 5′ 35″ | 5′ 40″ | —.14″ | 2976.85″ | 6.96″ | —.15″ |
| | I M | 5 45 | | | 7.08 | | |
| | II B | 5 34 | 5 41 | —.13 | 6.75 | 6.82 | —.29 |
| | II M | 5 48 | | | 6.88 | | |
| | III B | 5 42 | 5 46 | —.06 | 6.90 | 7.10 | —.01 |
| | III M | 5 49 | | | 7.31 | | |
| | IV B | 6 4 | 6 12 | +.31 | 7.55 | 7.57 | +.46 |
| | IV M | 6 21 | | | 7.59 | | |
| | | Means 24° 5′ 50″ ±5″ | | | | 2977.11″ ±.06″ | |
| 5 | II B | 35° 16′ 1″ | 16′ 6″ | —.27″ | 3431.49″ | 1.58″ | —.10″ |
| | II M | 16 11 | | | 1.67 | | |
| | III B | 16 42 | 16 43 | +.36 | 1.26 | 1.36 | —.32 |
| | III M | 16 44 | | | 1.45 | | |
| | IV B | 16 14 | 16 16 | —.10 | 2.02 | 2.10 | +.42 |
| | IV M | 16 18 | | | 2.17 | | |
| | | Means 35° 16′ 22″ ±4″ | | | | 3431.68″ ±.07″ | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B + M) | Δα sin π | π | ½(B + M) | Δπ |
|---|---|---|---|---|---|---|---|
| 6 | I B | 35° 58′ 41″ | 58′ 40″ | —″.30 | 3148″.36 | 8″.55 | +″.08 |
| | I M | 58 38 | | | 8.74 | | |
| | II B | 58 32 | 58 40 | —.30 | 7.66 | 8.16 | —.31 |
| | II M | 58 48 | | | 8.67 | | |
| | III B | 59 12 | 59 13 | +.20 | 8.24 | 8.05 | —.42 |
| | III M | 59 14 | | | 7.86 | | |
| | IV B | 59 30 | 59 28 | +.40 | 8.94 | 9.12 | +.65 |
| | IV M | 59 25 | | | 9.31 | | |
| | | Means 35° 59′ 0″ ±4″ | | | | 3148″.47 ±″.06 | |
| 7 | I B | 36° 25′ 11″ | 24′ 43″ | —″.02 | 2238″.16 | 8″.40 | +″.19 |
| | I M | 24 15 | | | 8.65 | | |
| | II B | 24 2 | 24 8 | —.41 | 7.88 | 7.93 | —.28 |
| | II M | 24 13 | | | 7.98 | | |
| | III B | 24 45 | 24 50 | +.06 | 8.36 | 8.24 | +.03 |
| | III M | 24 54 | | | 8.13 | | |
| | IV B | 25 26 | 25 18 | +.36 | 8.21 | 8.26 | +.05 |
| | IV M | 25 11 | | | 8.30 | | |
| | | Means 36° 24′ 45″ ±6″ | | | | 2238″.21 ±″.06 | |
| *8 | I B | 38° 13′ 3″ | 12′ 44″ | —″.06 | 2198″.22 | 8″.12 | +″.22 |
| | I M | 12 25 | | | 8.02 | | |
| | II B | 12 44 | 12 57 | +.09 | 7.82 | 7.87 | —.03 |
| | II M | 13 10 | | | 7.92 | | |
| | III B | 13 2 | 12 56 | +.08 | 7.92 | 8.05 | +.15 |
| | III M | 12 50 | | | 8.18 | | |
| | IV B | 12 53 | 12 38 | —.12 | 7.53 | 7.54 | —.36 |
| | IV M | 12 22 | | | 7.56 | | |
| | | Means 38° 12′ 49″ ±6″ | | | | 2197″.90 ±″.06 | |
| *9 | I B | 39° 5′ 31″ | 5′ 31″ | +″.26 | 1633″.51 | 3″.60 | +″.36 |
| | I M | 5 31 | | | 3.69 | | |
| | II B | 4 32 | 4 46 | —.10 | 3.03 | 3.16 | —.08 |
| | II M | 5 0 | | | 3.29 | | |
| | III B | 5 21 | 5 22 | +.19 | 3.41 | 3.32 | +.08 |
| | III M | 5 23 | | | 3.23 | | |
| | IV B | 4 16 | 4 14 | —.35 | 2.79 | 2.90 | —.34 |
| | IV M | 4 13 | | | 3.01 | | |
| | | Means 39° 4′ 58″ ±8″ | | | | 1633″.24 ±″.06 | |
| *10 | I B | 41° 38′ 11″ | 38′ 4″ | —″.14 | 3039″.58 | 9″.73 | +″.09 |
| | I M | 37 56 | | | 9.88 | | |
| | II B | 38 01 | 38 0 | —.20 | 9.63 | 9.83 | +.19 |
| | II M | 38 0 | | | 0.03 | | |
| | III B | 38 20 | 38 14 | +.02 | 9.54 | 9.64 | +.00 |
| | III M | 38 9 | | | 9.75 | | |
| | IV B | 38 34 | 38 33 | +.30 | 9.46 | 9.36 | —.28 |
| | IV M | 38 32 | | | 9.26 | | |
| | | Means 41° 38′ 13″ ±4″ | | | | 3039″.64 ±″.06 | |

### Table VII.—(*Continued.*) Mean Results.

| Star. | Plate. | $\alpha$ | ½$(B+M)$ | $\Delta\alpha \sin \pi$ | $\pi$ | ½$(B+M)$ | $\Delta\pi$ |
|---|---|---|---|---|---|---|---|
| *11 | I B | 44° 28′ 27″ | 28′ 14″ | —″.26 | 1427″.54 | 7″.60 | —″.20 |
| | I M | 28 2 | | | 7.67 | | |
| | II B | 27 51 | 28 16 | —.24 | 8.08 | 8.16 | +.36 |
| | II M | 28 40 | | | 8.24 | | |
| | III B | 28 38 | 28 40 | —.08 | 7.22 | 7.34 | —.46 |
| | III M | 28 43 | | | 7.47 | | |
| | IV B | 30 26 | 30 14 | +.58 | 8.19 | 8.08 | +.28 |
| | IV M | 30 3 | | | 7.96 | | |
| | | Means 44° 28′ 51″ ±9″ | | | | 1427″.80 ±″.06 | |
| *12 | I B | 57° 51′ 11″ | 51′ 0″ | +″.06 | 2939″.67 | 9″.92 | +″.26 |
| | I M | 50 49 | | | 0.16 | | |
| | II B | 50 44 | 50 49 | —.10 | 9.68 | 0.02 | +.36 |
| | II M | 50 54 | | | 0 37 | | |
| | III B | 51 8 | 51 14 | +.25 | 9.16 | 9.40 | —.26 |
| | III M | 51 20 | | | 9.63 | | |
| | IV B | 50 38 | 50 40 | —.22 | 9.42 | 9.32 | —.34 |
| | IV M | 50 42 | | | 9.22 | | |
| | | Means 57° 50′ 56″ ±5″ | | | | 2939″.66 ±″.06 | |
| 13 | I B | 58° 39′ 16″ | 38′ 49″ | —″.05 | 2653″.00 | 2″.72 | —″.07 |
| | I M | 38 22 | | | 2.45 | | |
| | II B | 38 51 | 38 52 | —.01 | 3.09 | 3.10 | +.31 |
| | II M | 38 53 | | | 3.11 | | |
| | III B | 38 45 | 38 48 | —.06 | 2.06 | 2.18 | —.61 |
| | III M | 38 51 | | | 2.30 | | |
| | IV B | 38 58 | 39 4 | +.14 | 3.26 | 3.15 | +.36 |
| | IV M | 39 9 | | | 3.04 | | |
| | | Means 58° 38′ 53″ ±5″ | | | | 2652″.79 ±″.06 | |
| *14 | I B | 63° 16′ 50″ | 16′ 44″ | +″.00 | 3614″.00 | 4″.19 | —″.01 |
| | I M | 16 38 | | | 4.38 | | |
| | II B | 17 2 | 16 56 | +.22 | 4.02 | 4.18 | —.02 |
| | II M | 16 50 | | | 4.34 | | |
| | III B | 16 56 | 16 53 | +.16 | 4.14 | 4.18 | —.02 |
| | III M | 16 50 | | | 4.21 | | |
| | IV B | 16 25 | 16 25 | —.34 | 4.01 | 4.27 | +.07 |
| | IV M | 16 25 | | | 4.53 | | |
| | | Means 63° 16′ 44″ ±4″ | | | | 3614″.20 ±″.06 | |
| 15 | I B | 63° 49′ 10″ | 48′ 46″ | —″.18 | 3211″.09 | 1″.14 | +″.13 |
| | I M | 48 22 | | | 1.20 | | |
| | II B | 48 55 | 49 3 | +.10 | 0.59 | 0.98 | —.03 |
| | II M | 49 11 | | | 1.36 | | |
| | IV B | 49 14 | 49 3 | +.10 | 0.72 | 0.92 | —.09 |
| | IV M | 48 52 | | | 1.12 | | |
| | | Means 63° 48′ 57″ ±5″ | | | | 3211″.01 ±″.08 | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B+M) | Δα sin π | π | ½(B+M) | Δπ |
|---|---|---|---|---|---|---|---|
| 16 | I B | 67° 39′ 55″ | 39′ 43″ | —″.27 | 3559″.63 | 9″.53 | +″.13 |
| | I M | 39 31 | | | 9.43 | | |
| | II B | 40 13 | 40 18 | +.32 | 9.74 | 9.44 | +.04 |
| | II M | 40 24 | | | 9.14 | | |
| | III B | 40 19 | 39 56 | —.05 | 8.91 | 9.23 | —.17 |
| | III M | 39 34 | | | 9.55 | | |
| | | Means 67° 39′ 59″ ± 4″ | | | | | 3559″.40 ±″.07 |
| 17 | I B | 68° 52′ 19″ | 52′ 12″ | —″.02 | 3276″.41 | 6″.50 | —″.23 |
| | I M | 52 4 | | | 6.60 | | |
| | III B | 52 35 | 52 22 | +.14 | 6.57 | 6.77 | —.06 |
| | III M | 52 10 | | | 6.97 | | |
| | IV B | 52 3 | 52 4 | —.14 | 7.45 | 7.22 | +.39 |
| | IV M | 52 6 | | | 6.98 | | |
| | | Means 68° 52′ 13″ ± 5″ | | | | | 3276″.83 ±″.08 |
| 18 | I B | 70° 48′ 39″ | 48′ 14″ | —″.05 | 1238″.88 | 8″.94 | —″.05 |
| | I M | 47 48 | | | 9.01 | | |
| | II B | 49 1 | 48 58 | +.21 | 8.68 | 8.84 | —.15 |
| | II M | 48 55 | | | 9.00 | | |
| | III B | 48 48 | 48 26 | +.02 | 8.59 | 8.55 | —.44 |
| | III M | 48 3 | | | 8.51 | | |
| | IV B | 48 15 | 47 55 | —.17 | 9.66 | 9.64 | +.65 |
| | IV M | 47 35 | | | 9.61 | | |
| | | Means 70° 48′ 23″ ± 11″ | | | | | 1238″.99 ±″.06 |
| *19 | I B | 75° 0′ 56″ | 0′ 34″ | —″.14 | 2360″.10 | 0″.06 | —″.06 |
| | I M | 0 12 | | | 0.03 | | |
| | II B | 1 2 | 0 58 | +.12 | 0.13 | 0.13 | +.01 |
| | II M | 0 55 | | | 0 13 | | |
| | III B | 0 44 | 0 50 | +.03 | 9.82 | 9.92 | —.20 |
| | III M | 0 57 | | | 0.01 | | |
| | IV B | 0 58 | 0 46 | —.01 | 0.27 | 0.35 | +.23 |
| | IV M | 0 33 | | | 0.43 | | |
| | | Means 75° 0′ 47″ ± 6″ | | | | | 2360″.12 ±″.06 |
| 20 | I B | 83° 5′ 4″ | —″.12 | 4′ 52″ | 2178″.63 | 8″.60 | —″.75 |
| | I M | 4 41 | | | 8.57 | | |
| | IV B | 4 23 | +.12 | 4 30 | 0.19 | 0.10 | +.75 |
| | IV M | 4 38 | | | 0.00 | | |
| | | Means 83° 4′ 41″ ± 9″ | | | | | 2179″.35 ±″.10 |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B + M) | Δα sin π | π | ½(B + M) | Δπ |
|---|---|---|---|---|---|---|---|
| | | ° ′ ″ | ′ ″ | ″ | ″ | ″ | ″ |
| *21 | I B | 92 21 54 | 21 19 | —.07 | 1887.57 | 7.52 | +.33 |
| | I M | 20 44 | | | 7.48 | | |
| | II B | 22 02 | 21 53 | +.23 | 6.86 | 7.10 | —.09 |
| | II M | 21 44 | | | 7.34 | | |
| | III B | 21 4 | 21 0 | —.24 | 6.96 | 6.96 | —.23 |
| | III M | 20 57 | | | 6.95 | | |
| | IV B | 21 43 | 21 36 | +.08 | 7.05 | 7.17 | —.02 |
| | IV M | 21 30 | | | 7.29 | | |
| | | | Means 92° 21′ 27″ ±7″ | | | 1887″.19 ±″.06 | |
| 22 | I B | 98° 0 37 | 0 31 | +.22 | 3263.33 | 3.51 | +.43 |
| | I M | 0 25 | | | 3.69 | | |
| | II B | 0 21 | 0 8 | —.14 | 3.01 | 3.16 | +.08 |
| | II M | 59 54 | | | 3.30 | | |
| | III B | 0 39 | 0 38 | +.32 | 2.54 | 2.50 | —.58 |
| | III M | 0 37 | | | 2.45 | | |
| | IV B | 0 2 | 59 53 | —.38 | 2.99 | 3.14 | +.06 |
| | IV M | 59 44 | | | 3.30 | | |
| | | | Means 98° 0′ 17″ ±4″ | | | 3263″.08 ±″.06 | |
| *23 | I B | 100° 2 59 | 2 21 | —.20 | 809.31 | 9.22 | +.09 |
| | I M | 1 43 | | | 9.12 | | |
| | II B | 2 43 | 2 20 | —.21 | 9.27 | 9.36 | +.23 |
| | II M | 1 56 | | | 9.45 | | |
| | III B | 2 38 | 2 26 | —.18 | 9.00 | 9.03 | —.10 |
| | III M | 2 15 | | | 9.06 | | |
| | IV B | 6 11 | 5 42 | +.60 | 8.91 | 8.90 | —.23 |
| | IV M | 5 13 | | | 8.90 | | |
| | | | Means 100° 3′ 12″ ±16″ | | | 809″.13 ±″.06 | |
| *24 | I B | 100 44 45 | 44 50 | —.22 | 3444.60 | 4.52 | +.18 |
| | I M | 44 55 | | | 4.44 | | |
| | II B | 45 06 | 45 8 | +.08 | 4.73 | 4.74 | +.40 |
| | II M | 45 10 | | | 4.76 | | |
| | III B | 44 58 | 44 58 | —.08 | 3.40 | 3.84 | —.50 |
| | III M | 44 58 | | | 4.28 | | |
| | IV B | 45 15 | 45 16 | +.22 | 4.36 | 4.25 | —.09 |
| | IV M | 45 18 | | | 4.14 | | |
| | | | Means 100° 45′ 3″ ±4″ | | | 3444″.34 ±″.06 | |
| *25 | I B | 120 19 53 | 19 26 | +.12 | 1530.95 | 0.92 | —.22 |
| | I M | 18 59 | | | 0.90 | | |
| | II B | 19 59 | 19 47 | +.27 | 1.20 | 1.34 | +.20 |
| | II M | 19 35 | | | 1.48 | | |
| | III B | 19 39 | 19 14 | +.04 | 1.06 | 1.18 | +.04 |
| | III M | 18 49 | | | 1.30 | | |
| | IV B | 18 14 | 18 10 | —.41 | 1.22 | 1.14 | —.00 |
| | IV M | 18 5 | | | 1.06 | | |
| | | | Means 120° 19′ 9″ ±9″ | | | 1531″.14 ±″.06 | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B + M) | Δα sin π | π | ½(B + M) | Δπ |
|---|---|---|---|---|---|---|---|
| *26 | I B | 133° 35′ 23″ | 34′ 56″ | +″.03 | 1298″.27 | 8″.24 | —″.04 |
| | I M | 34 29 | | | 8.20 | | |
| | II B | 36 2 | 35.24 | +.20 | 8.49 | 8.58 | +.36 |
| | II M | 34 47 | | | 8.66 | | |
| | III B | 34 41 | 34.26 | —.15 | 7.82 | 7.88 | —.40 |
| | III M | 34 10 | | | 7.95 | | |
| | IV B | 34 40 | 34.38 | —.08 | 8.48 | 8.42 | +.14 |
| | IV M | 34 37 | | | 8.35 | | |
| | | Means 133° 34′ 51″ ±11″ | | | | 1298″.28 ±″.06 | |
| 27 | I B | 154° 5′ 47″ | 5′ 57″ | +″.37 | 2014″.49 | 4″.39 | +″.13 |
| | I M | 6 7 | | | 4.29 | | |
| | II B | 5 17 | 5 22 | +.02 | 4.30 | 4.44 | +.18 |
| | II M | 5 28 | | | 4.58 | | |
| | IV B | 4 39 | 4 40 | —.40 | 4.03 | 3.94 | —.32 |
| | IV M | 4 40 | | | 3.84 | | |
| | | Means 154° 5′ 20″ ± 8″ | | | | 2014″.26 ±″.08 | |
| *28 | I B | 163° 25′ 42″ | 25′ 34″ | +″.06 | 2301″.82 | 1″.58 | +″.41 |
| | I M | 25 27 | | | 1.35 | | |
| | II B | 26 19 | 25 54 | +.28 | 1.25 | 1.32 | +.15 |
| | II M | 25 29 | | | 1.39 | | |
| | III B | 24 35 | 24 42 | —.52 | 1.04 | 1.03 | —.14 |
| | III M | 24 48 | | | 1.02 | | |
| | IV B | 25 46 | 25 46 | +.19 | 0.76 | 0.76 | —.41 |
| | IV M | 25 46 | | | 0.76 | | |
| | | Means 163° 25′ 29″ ± 6″ | | | | 2301″.17 ±″.06 | |
| *29 | I B | 171° 19′ 21″ | 19′ 16″ | +″.13 | 2260″.39 | 0″.24 | +″.33 |
| | I M | 19 10 | | | 0.09 | | |
| | II B | 19 11 | 19 11 | +.08 | 0 15 | 0.26 | +.35 |
| | II M | 19 11 | | | 0.37 | | |
| | III B | 19 27 | 19 30 | +.29 | 9.45 | 9.60 | —.31 |
| | III M | 19 32 | | | 9.74 | | |
| | IV B | 18 26 | 18 20 | —.48 | 9.61 | 9.55 | —.36 |
| | IV M | 18 15 | | | 9.49 | | |
| | | Means 171° 19′ 4″ ± 6″ | | | | 2259″.91 ±″.06 | |
| *30 | I B | 172° 40′ 36″ | 39′ 56″ | +″.41 | 1617″.85 | 7″.63 | —″.05 |
| | I M | 39 16 | | | 7.41 | | |
| | II B | 39 12 | 38 46 | —.15 | 7.51 | 7.50 | —.18 |
| | II M | 38 20 | | | 7.50 | | |
| | III B | 39 15 | 38 52 | —.10 | 7.94 | 7.72 | +.04 |
| | III M | 38 28 | | | 7.49 | | |
| | IV B | 38 42 | 38 46 | —.15 | 7.92 | 7.88 | +.20 |
| | IV M | 38 50 | | | 7.83 | | |
| | | Means 172° 39′ 5″ ± 8″ | | | | 1617″.68 ±″.06 | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B+M) | Δα sin π | π | B½(+M) | Δπ |
|---|---|---|---|---|---|---|---|
| 31 | I B | 180° 26′ 40″ | 26′ 55″ | +.″30 | 2687.″00 | 7.″10 | —.″24 |
| | I M | 27 10 | | | 7.20 | | |
| | II B | 26 54 | 26 30 | —.03 | 7.05 | 6.94 | —.40 |
| | II M | 26 5 | | | 6.83 | | |
| | III B | 26 48 | 27 0 | +.36 | 7.20 | 7.50 | +.16 |
| | III M | 27 11 | | | 7.81 | | |
| | IV B | 25 53 | 25 41 | —.68 | 7.94 | 7.82 | +.48 |
| | IV M | 25 29 | | | 7.69 | | |
| | | Means 180° 26′ 32″ ±5″ | | | 2687.″34 ±.″06 | | |
| *32 | I B | 191° 13′ 3″ | 13′ 2″ | —.″33 | 2067.″36 | 7.″28 | +.″11 |
| | I M | 13 0 | | | 7.19 | | |
| | II B | 14 0 | 13 55 | +.20 | 6.98 | 7.00 | —.17 |
| | II M | 13 50 | | | 7.02 | | |
| | III B | 13 27 | 13 31 | —.04 | 7.35 | 7.36 | +.19 |
| | III M | 13 35 | | | 7.38 | | |
| | IV B | 13 42 | 13 51 | +.16 | 7.04 | 7.04 | —.13 |
| | IV M | 14 0 | | | 7.05 | | |
| | | Means 191° 13′ 35″ ±6″ | | | 2067.″17 ±.″06 | | |
| 33 | I B | 191° 16′ 35″ | | —.″17 | 2340.″84 | | +.″18 |
| | II B | 16 54 | | +.06 | 0.65 | | —.01 |
| | III B | 17 12 | | +.28 | 0.67 | | +.01 |
| | IV B | 16 34 | | —.18 | 0.48 | | —.18 |
| | | Means 191° 16′ 49″ ±8″ | | | 2440.″66 ±.″09 | | |
| 34 | I B | 191° 21′ 35″ | 21′ 52″ | —.″17 | 2548.″33 | 8.″27 | +.″41 |
| | I M | 22 8 | | | 8.21 | | |
| | II B | 22 32 | 22 18 | +.14 | 8.44 | 8.05 | +.19 |
| | II M | 22 3 | | | 7.66 | | |
| | III B | 21 53 | 21 50 | —.19 | 7.93 | 7.88 | +.02 |
| | III M | 21 47 | | | 7.82 | | |
| | IV B | 22 41 | 22 25 | +.23 | 7.45 | 7.24 | —.62 |
| | IV M | 22 9 | | | 7.02 | | |
| | | Means 191° 22′ 6″ ±5″ | | | 2547.″86 ±.″06 | | |
| *35 | I B | 195° 35′ 0″ | 35′ 18″ | —.″02 | 2274.″53 | 4.″51 | +.″45 |
| | I M | 35 35 | | | 4.49 | | |
| | II B | 35 16 | 35 3 | —.19 | 4.45 | 4.38 | +.32 |
| | II M | 34 50 | | | 4.30 | | |
| | III B | 35 19 | 35 18 | —.02 | 4.04 | 3.91 | —.15 |
| | III M | 35 16 | | | 3.78 | | |
| | IV B | 35 44 | 35 39 | +.21 | 3.52 | 3.46 | —.60 |
| | IV M | 35 34 | | | 3.39 | | |
| | | Means 195° 35′ 20″ ±6″ | | | 2274.″06 ±.″06 | | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | $\alpha$ | ½($B$+$M$) | $\Delta\alpha \sin \pi$ | $\pi$ | $B$½(+$M$) | $\Delta\pi$ |
|---|---|---|---|---|---|---|---|
| 36 | II B | 196° 53′ 17″ | 52′ 56″ | +″.20 | 862″.39 | 2″.30 | +″.34 |
| | II M | 52 36 | | | 2.21 | | |
| | III B | 51 52 | 51 52 | —.06 | 2.42 | 2.27 | +.31 |
| | III M | 51 52 | | | 2.12 | | |
| | IV B | 51 34 | 51 30 | —.14 | 1.38 | 1.32 | —.64 |
| | IV M | 51 25 | | | 1.27 | | |
| | | Means 196° 52′ 6″ ±18″ | | | | 861″.96 ±″.07 | |
| 37 | II B | 201° 42′ 56″ | 44′ 57″ | —″.04 | 811″.22 | 1″.16 | +″.07 |
| | II M | 46 58 | | | 1.10 | | |
| | IV B | 44 27 | 45 20 | +.05 | 0.72 | 1.02 | —.07 |
| | IV M | 46 13 | | | 1.33 | | |
| | | Means 201° 45′ 8″ ±23″ | | | | 811″.09 ±″.09 | |
| *38 | I B | 211° 36′ 27″ | 36′ 20″ | +″.06 | 1657″.90 | 7″.76 | —″.27 |
| | I M | 36 12 | | | 7.61 | | |
| | II B | 37 2 | 36 40 | +.22 | 8.18 | 8.20 | +.17 |
| | II M | 36 17 | | | 8.22 | | |
| | III B | 35 55 | 35 52 | —.16 | 8.23 | 8.10 | +.07 |
| | III M | 35 50 | | | 7.97 | | |
| | IV B | 35 52 | 35 58 | —.11 | 8.12 | 8.06 | +.03 |
| | IV M | 36 4 | | | 7.99 | | |
| | | Means 211° 36′ 12″ ± 8″ | | | | 1658″.03 ±″.06 | |
| *39 | I B | 215° 48′ 41″ | 49′ 0″ | +″.24 | 2186″.19 | 5″.69 | +″.05 |
| | I M | 49 19 | | | 5.19 | | |
| | II B | 48 59 | 48 55 | +.19 | 5.49 | 5.12 | —.52 |
| | II M | 48 51 | | | 4.76 | | |
| | III B | 48 42 | 48 18 | —.22 | 5.77 | 5.56 | —.08 |
| | III M | 47 53 | | | 5.35 | | |
| | IV B | 48 15 | 48 17 | —.23 | 6.08 | 6.20 | +.56 |
| | IV M | 48 19 | | | 6.31 | | |
| | | Means 215° 48′ 38″ ± 6″ | | | | 2185″.64 ±″.06 | |
| 40 | I B | 217° 21′ 23″ | 21′ 48″ | +″.06 | 2426″.12 | 6″.14 | +″.02 |
| | I M | 22 12 | | | 6.16 | | |
| | II B | 21 46 | 21 25 | —.26 | 6.08 | 5.94 | +.06 |
| | II M | 21 4 | | | 5.81 | | |
| | III B | 21 53 | 22 2 | +.23 | 5.87 | 5.66 | —.46 |
| | III M | 22 10 | | | 5.45 | | |
| | IV B | 21 44 | 21 54 | +.13 | 5.45 | 5.76 | —.36 |
| | IV M | 22 4 | | | 6.07 | | |
| | | Means 217° 21′ 47″ ± 5″ | | | | 2425″.88 ±″.06 | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B + M) | Δα sin π | π | ½(B + M) | Δπ |
|---|---|---|---|---|---|---|---|
| *41 | I B | 229° 24′ 30″ | 24′ 9″ | —″.32 | 1374″.50 | 4″.47 | +″.29 |
| | I M | 23 48 | | | 4.44 | | |
| | II B | 25 34 | 25 21 | +.19 | 4.67 | 4.66 | +.48 |
| | II M | 25 8 | | | 4.66 | | |
| | III B | 24 45 | 24 59 | +.04 | 3.97 | 3.78 | —.40 |
| | III M | 25 13 | | | 3.60 | | |
| | IV B | 25 12 | 25 9 | +.08 | 3.97 | 3.80 | —.38 |
| | IV M | 25 6 | | | 3.63 | | |
| | | Means 229° 24′ 54″ ±9″ | | | 1374″.18 ±″.06 | | |
| *42 | I B | 237° 21′ 26″ | 21′ 32″ | +″.04 | 1153″.88 | 3″.88 | +″.03 |
| | I M | 21 38 | | | 3.88 | | |
| | II B | 22 6 | 21 58 | +.20 | 3.80 | 3.69 | —.16 |
| | II M | 21 51 | | | 3.58 | | |
| | III B | 22 7 | 21 58 | +.20 | 3.58 | 3.52 | —.33 |
| | III M | 21 50 | | | 3.46 | | |
| | IV B | 20 7 | 20 12 | —.44 | 4.33 | 4.30 | +.45 |
| | IV M | 20 18 | | | 4.27 | | |
| | | Means 237° 21′ 25″ ±11″ | | | 1153″.85 ±″.06 | | |
| *43 | I B | 238° 14′ 15″ | 14′ 13″ | —″.17 | 1912″.98 | 2″.76 | —″.06 |
| | I M | 14 11 | | | 2.53 | | |
| | II B | 14 34 | 14 12 | —.18 | 3.19 | 3.10 | +.28 |
| | II M | 13 49 | | | 3.01 | | |
| | III B | 14 47 | 15 4 | +.29 | 2.90 | 2.80 | —.02 |
| | III M | 15 20 | | | 2.70 | | |
| | IV B | 14 48 | 14 40 | +.07 | 2.66 | 2.60 | —.22 |
| | IV M | 14 33 | | | 2.53 | | |
| | | Means 238° 14′ 32″ ±7″ | | | 1912″.82 ±″.06 | | |
| *44 | I B | 244° 33′ 53″ | 34′ 20″ | —″.20 | 2811″.50 | 1″.25 | +″.05 |
| | I M | 34 48 | | | 1.00 | | |
| | II B | 34 51 | 34 56 | +.31 | 0.68 | 0.95 | —.20 |
| | II M | 35 0 | | | 1.22 | | |
| | III B | 34 49 | 34 49 | +.21 | 1 62 | 1.62 | +.47 |
| | IV B | 34 18 | 34 10 | —.34 | 0.87 | 0.84 | —.31 |
| | IV M | 34 1 | | | 0.80 | | |
| | | Means 244° 34′ 34″ ±5″ | | | 2811″.15 ±″.06 | | |
| *45 | I B | 254° 56′ 39″ | 56′ 34″ | +″.33 | 731″.97 | 1″.94 | +″.08 |
| | I M | 56 30 | | | 1.91 | | |
| | II B | 55 15 | 54 38 | —.14 | 1.80 | 1.54 | —.32 |
| | II M | 54 0 | | | 1.27 | | |
| | III B | 53 31 | 53 48 | —.34 | 1.81 | 1.80 | —.06 |
| | III M | 54 5 | | | 1.78 | | |
| | IV B | 55 28 | 55 48 | +.14 | 2.21 | 2.18 | +.32 |
| | IV M | 56 9 | | | 2.15 | | |
| | | Means 254° 55′ 12″ ±16″ | | | 731″.86 ±″.06 | | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B + M) | Δα sin π | π | ½(B + M) | Δπ |
|---|---|---|---|---|---|---|---|
| *46 | I B | 262° 59′ 3″ | 58′ 58″ | +″.16 | 1230″.59 | 0″.59 | +″.16 |
| | I M | 58 53 | | | 0.59 | | |
| | II B | 57 38 | 57 36 | —.34 | 0.30 | 0.28 | —.15 |
| | II M | 57 33 | | | 0.26 | | |
| | III B | 57 18 | 57 32 | —.36 | 0.53 | 0.36 | —.07 |
| | III M | 57 46 | | | 0.19 | | |
| | IV B | 59 37 | 0 0 | +.53 | 0.48 | 0.50 | +.07 |
| | IV M | 0 23 | | | 0.53 | | |
| | | Means 262° 58′ 32″ ±11″ | | | | 1230″.43 ±″.06 | |
| *47 | I B | 263° 50′ 56″ | 51′ 23″ | +″.05 | 3212″.68 | 2″.69 | +″.14 |
| | I M | 51 50 | | | 2.70 | | |
| | II B | 51 36 | 51 36 | +.26 | 1.85 | 2.05 | —.50 |
| | II M | 51 35 | | | 2.25 | | |
| | III B | 50 37 | 50 55 | —.40 | 2.62 | 2.77 | +.22 |
| | III M | 51 13 | | | 2.92 | | |
| | IV B | 51 45 | 51 28 | +.13 | 2.76 | 2.68 | +.13 |
| | IV M | 51 12 | | | 2.59 | | |
| | | Means 263° 51′ 20″ ± 4″ | | | | 3212″55 ±″.06 | |
| 48 | I B | 265° 28′ 52″ | 29′ 41″ | +″.01 | 328″.48 | 8″.42 | —″.23 |
| | I M | 30 30 | | | 8.36 | | |
| | II B | 25 54 | 26 6 | —.33 | 8.74 | 8.72 | +.07 |
| | II M | 26 18 | | | 8.70 | | |
| | III B | 27 30 | 29 41 | +.01 | 8.70 | 8.53 | —.12 |
| | III M | 31 52 | | | 8.36 | | |
| | IV B | 33 45 | 32 38 | +.30 | 8.82 | 8.94 | +.29 |
| | IV M | 31 32 | | | 9.07 | | |
| | | Means 265° 29′ 32″ ±40″ | | | | 328″.65 ±″.06 | |
| *49 | I B | 266° 12′ 43″ | 14′ 30″ | +″.18 | 339″.98 | 9″.94 | —″.13 |
| | I M | 16 16 | | | 9.90 | | |
| | II B | 8 16 | 9 28 | —.30 | 0.30 | 0.27 | +.20 |
| | II M | 10 40 | | | 0.24 | | |
| | III B | 10 54 | 11 29 | —.11 | 9.51 | 9.48 | —.59 |
| | III M | 12 4 | | | 9.45 | | |
| | IV B | 15 9 | 15 2 | +.23 | 0.54 | 0.58 | +.51 |
| | IV M | 14 56 | | | 0.61 | | |
| | | Means 266° 12′ 37″ ±40″ | | | | 340″.07 ±″.06 | |
| *50 | I B | 268° 28′ 39″ | 28′ 47″ | —″.04 | 2978″.16 | 8″.06 | +″.16 |
| | I M | 28 55 | | | 7.96 | | |
| | II B | 28 41 | 28 40 | —.14 | 7.89 | 8.00 | +.10 |
| | II M | 28 39 | | | 8.11 | | |
| | III B | 28 40 | 28 41 | —.13 | 8.16 | 7.90 | +.00 |
| | III M | 28 42 | | | 7.63 | | |
| | IV B | 29 10 | 29 14 | +.34 | 7.79 | 7.66 | —.24 |
| | IV M | 29 18 | | | 7.52 | | |
| | | Means 268° 28′ 50″ ± 5″ | | | | 2977″.90 ±″.06 | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B+M) | Δα sin π | π | ½(B+M) | Δπ |
|---|---|---|---|---|---|---|---|
| 51 | I B | 277° 1′ 46″ | 1′ 54″ | —″.03 | 1759″.12 | 9″.06 | +″.14 |
| | I M | 2 2 | | | 9.00 | | |
| | II B | 1 5 | 1 2 | —.55 | 8.74 | 8.78 | —.14 |
| | II M | 0 58 | | | 8.82 | | |
| | III B | 1 45 | 1 42 | —.13 | 9.15 | 9.04 | +.12 |
| | III M | 1 40 | | | 8.94 | | |
| | IV B | 3 8 | 3 14 | +.62 | 8.84 | 8.80 | —.12 |
| | IV M | 3 19 | | | 8.76 | | |
| | | Means 277° 1′ 58″ ± 8″ | | | | 1758″.92 ±″.06 | |
| *52 | I B | 281° 35′ 39″ | 36′ 4″ | —″.20 | 2622″.07 | 1″.96 | +″.04 |
| | I M | 36 30 | | | 1.84 | | |
| | II B | 36 40 | 36 23 | +.05 | 1.99 | 2.04 | +.12 |
| | II M | 36 6 | | | 2.08 | | |
| | III B | 35 58 | 36 1 | —.23 | 1.82 | 1.78 | —.14 |
| | III M | 36 4 | | | 1.75 | | |
| | IV B | 36 40 | 36 48 | +.38 | 2.03 | 1.89 | —.03 |
| | IV M | 36 56 | | | 1.75 | | |
| | | Means 281° 36′ 19″ ± 5″ | | | | 2621″.92 ±″.06 | |
| *53 | I B | 281° 36′ 29″ | 36′ 44″ | +″.18 | 1212″.17 | 2″.08 | +″.26 |
| | I M | 36 58 | | | 2.00 | | |
| | II B | 36 6 | 35 47 | —.16 | 2 04 | 2.08 | +.26 |
| | II M | 35 28 | | | 2.13 | | |
| | III B | 35 53 | 36 6 | —.05 | 1.54 | 1.56 | —.26 |
| | III M | 36 18 | | | 1.57 | | |
| | IV B | 36 18 | 36 18 | +.02 | 1.53 | 1.58 | —.24 |
| | IV M | 36 19 | | | 1.62 | | |
| | | Means 281° 36′ 14″ ±11″ | | | | 1211″.82 ±″.06 | |
| *54 | I B | 285° 41′ 41″ | 42′ 0″ | +″.00 | 2430″.53 | 0″.60 | —″.01 |
| | I M | 42 18 | | | 0.67 | | |
| | II B | 41 52 | 41 44 | —.19 | 0.67 | 0.65 | +.04 |
| | II M | 41 36 | | | 0.63 | | |
| | III B | 42 0 | 42 4 | +.05 | 0.68 | 0.50 | —.11 |
| | III M | 42 9 | | | 0.31 | | |
| | IV B | 42 14 | 42 14 | +.17 | 0.77 | 0.70 | +.09 |
| | IV M | 42 13 | | | 0.62 | | |
| | | Means 285° 42′ 0″ ± 5″ | | | | 2430″.61 ±″.06 | |
| 55 | I B | 291° 37′ 0″ | 36′ 41″ | —″.12 | 502″.33 | 2″.38 | —″.11 |
| | I M | 36 22 | | | 2.44 | | |
| | IV B | 38 58 | 38 39 | +.12 | 2.61 | 2.60 | +.11 |
| | IV M | 38 20 | | | 2.59 | | |
| | | Means 291° 37′ 40″ ±52″ | | | | 502″.49 ±″.10 | |

TABLE VII.—(*Continued.*) MEAN RESULTS.

| Star. | Plate. | $a$ | ½($B$+$M$) | $\Delta a$ sin $\pi$ | $\pi$ | ½($B$+$M$) | $\Delta\pi$ |
|---|---|---|---|---|---|---|---|
| *56 | I B | 292° 6′ 21″ | 6′ 34″ | —″.14 | 1593″.74 | 3″.58 | +″.10 |
| | I M | 6 46 | | | 3.43 | | |
| | II B | 7 1 | 7 8 | +.13 | 3.64 | 3.71 | +.23 |
| | II M | 7 15 | | | 3.78 | | |
| | III B | 7 9 | 7 16 | +.19 | 3.72 | 3.66 | +.18 |
| | III M | 7 23 | | | 3.59 | | |
| | IV B | 6 30 | 6 29 | —.18 | 2.93 | 2.95 | —.53 |
| | IV M | 6 28 | | | 2.97 | | |
| | | Means 292° 6′ 52″ ± 8″ | | | | 1593″.48 ±″.06 | |
| 57 | I B | 293° 38′ 58″ | 40′ 18″ | +″.26 | 1055″.57 | 5″.65 | +″.13 |
| | I M | 41 37 | | | 5.73 | | |
| | II B | 38 53 | 39 2 | —.12 | 5.85 | 5.92 | +.40 |
| | II M | 39 10 | | | 5.99 | | |
| | IV B | 38 47 | 39 0 | —.14 | 5.02 | 5.00 | —.52 |
| | IV M | 39 13 | | | 4.99 | | |
| | | Means 293° 39′ 27″ ±15″ | | | | 1055″.52 ±″.08 | |
| *58 | I B | 297° 14′ 11″ | 14′ 13″ | —″.34 | 1291″.68 | 1″.52 | +″.28 |
| | I M | 14 15 | | | 1.37 | | |
| | II B | 15 45 | 15 34 | +.15 | 1.23 | 1.28 | +.04 |
| | II M | 15 23 | | | 1.34 | | |
| | III B | 14 34 | 15 8 | —.01 | 1.16 | 1.21 | —.03 |
| | III M | 15 43 | | | 1.26 | | |
| | IV B | 15 21 | 15 40 | +.19 | 1.01 | 0.96 | —.28 |
| | IV M | 15 59 | | | 0.90 | | |
| | | Means 297° 15′ 9″ ±11″ | | | | 1291″.24 ±″.06 | |
| 59 | I B | 307° 21′ 37″ | 21′ 14″ | —″.37 | 1542″.28 | 2″.08 | +″.24 |
| | I M | 20 51 | | | 1.89 | | |
| | IV B | 22 40 | 22 45 | +.36 | 1.77 | 1 60 | —.24 |
| | IV M | 22 50 | | | 1.44 | | |
| | | Means 307° 22′ 0″ ±13″ | | | | 1541″.84 ±″.10 | |
| *60 | I B | 311° 8′ 32″ | 8′ 54″ | +″.10 | 851″.70 | 1″.71 | +″.10 |
| | I M | 9 15 | | | 1.72 | | |
| | II B | 6 14 | 6 48 | —.40 | 1.88 | 1.79 | +.18 |
| | II M | 7 23 | | | 1.70 | | |
| | III B | 7 34 | 7 40 | —.19 | 1.77 | 1.69 | +.08 |
| | III M | 7 47 | | | 1.61 | | |
| | IV B | 10 20 | 10 30 | +.49 | 1.17 | 1.24 | —.37 |
| | IV M | 10 39 | | | 1.30 | | |
| | | Means 311° 8′ 28″ ±16″ | | | | 851″.61 ±″.06 | |

TABLE VII.—(*Concluded.*) MEAN RESULTS.

| Star. | Plate. | α | ½(B + M) | Δα sin π | π | ½(B + M) | Δπ |
|---|---|---|---|---|---|---|---|
| 61 | I B | 314° 53′ 20″ | 53′ 34″ | +″.06 | 1581″.43 | 1″.51 | —″.01 |
| | I M | 53 47 | | | 1.59 | | |
| | II B | 52 57 | 52 40 | —.38 | 1.41 | 1.44 | —.08 |
| | II M | 52 24 | | | 1.47 | | |
| | III B | 52 56 | 52 50 | —.30 | 1.53 | 1.43 | —.09 |
| | III M | 52 44 | | | 1.33 | | |
| | IV B | 54 38 | 54 44 | +.62 | 1.81 | 1.72 | +.20 |
| | IV M | 54 51 | | | 1.64 | | |
| | | Means 314° 53′ 27″ ± 8″ | | | 1581″.52 ±″.06 | | |
| *62 | I B | 315° 28′ 5″ | 28′ 20″ | —″.11 | 1353″.58 | 3″.62 | +″.06 |
| | I M | 28 34 | | | 3.67 | | |
| | II B | 29 23 | 29 5 | +.20 | 3.36 | 3.55 | —.01 |
| | II M | 28 47 | | | 3.74 | | |
| | III B | 28 45 | 29 6 | +.21 | 3.58 | 3.56 | +.00 |
| | III M | 29 26 | | | 3.55 | | |
| | IV B | 27 53 | 27 54 | —.29 | 3.51 | 3 49 | —.07 |
| | IV M | 27 55 | | | 3.47 | | |
| | | Means 315° 28′ 36″ ± 9″ | | | 1353″.56 ±″.06 | | |
| *63 | I B | 317° 45′ 37″ | 45′ 50″ | —″.37 | 3252″.89 | 3″.02 | +″.16 |
| | I M | 46 3 | | | 3.14 | | |
| | II B | 46 22 | 46 36 | +.37 | 2.96 | 2.94 | +.08 |
| | II M | 46 49 | | | 2.92 | | |
| | III B | 46 7 | 46 18 | +.08 | 3.09 | 2.91 | +.05 |
| | III M | 46 30 | | | 2.73 | | |
| | IV B | 46 8 | 46 7 | —.10 | 2.57 | 2.56 | —.30 |
| | IV M | 46 6 | | | 2.55 | | |
| | | Means 317° 46′ 13″ ± 4″ | | | 3252″.86 ±″.06 | | |
| *64 | I B | 322° 6′ 54″ | 6′ 56″ | —″.10 | 1186″.00 | 5″.98 | —″.02 |
| | I M | 6 59 | | | 5.96 | | |
| | II B | 6 5 | 6 30 | +.26 | 6.06 | 5.95 | —.05 |
| | II M | 6 55 | | | 5.84 | | |
| | III B | 6 45 | 6 42 | —.19 | 5.78 | 5.86 | —.14 |
| | III M | 6 40 | | | 5.93 | | |
| | IV B | 8 41 | 8 45 | +.55 | 6.20 | 6.21 | +.21 |
| | IV M | 8 49 | | | 6.22 | | |
| | | Means 322° 7′ 13″ ±11″ | | | 1186″.00 ±″.06 | | |
| *65 | I B | 348° 36′ 6″ | 36′ 18″ | +″.00 | 2904″.74 | 4″.77 | +″.07 |
| | I M | 36 30 | | | 4.80 | | |
| | II B | 35 52 | 36 7 | —.15 | 4.80 | 4.88 | +.18 |
| | II M | 36 22 | | | 4.95 | | |
| | III B | 36 10 | 36 18 | +.00 | 4.62 | 4.70 | +.00 |
| | III M | 36 26 | | | 4.77 | | |
| | IV B | 36 38 | 36 30 | +.17 | 4.03 | 4.44 | —.26 |
| | IV M | 36 23 | | | 4.86 | | |
| | | Means 348° 36′ 18″ ± 5″ | | | 2904″.70 ±″.06 | | |

## IX.

## Final Results.

The table of final results contains the magnitudes of the stars, their numbers in the Bonn Durchmusterung, and in Carrington's Redhill Catalogue, their right ascensions, north polar distances and declinations for 1888.0, and their rectangular coördinates for this epoch.

The magnitudes of Carrington's catalogue were adopted for such of his stars as are included in this publication, and were also taken as the standard of comparison for the remaining stars. They were determined by him with considerable care, as will be seen by a reference to his own description.* The faintest star recorded by him is $10^m.3$, which stands for $9^m.5$ on Argelander's Durchmusterung scale.

The magnitudes of the remaining stars were determined by visual comparisons made with the 12-in. equatorial of the Vassar College Observatory, March 21–23, 1900. This telescope has frequently been used for asteroid observations in which the *DM* stars are used for purposes of identification, so that the Argelander scale is familiar to the observers. Accordingly estimates were made on the *DM* scale, and $0^m.8$ added to reduce to the Carrington scale. Some slight variations in magnitude from the 1855 determinations were noticed, one of which was particularly interesting.

Star 3546 of Carrington's Catalogue, *DM* 89°.24 shows no trace on any one of the four photographic plates. It is not even included among the stars which were visible but too faint to be measured. It was observed by Carrington on four different nights and estimated at $10^m.3$ each time. It was seen with the Vassar telescope on the above dates, and was estimated at $9^m.6$ on *DM* scale, or $10^m.4$ on Carrington's scale. These facts seem to indicate a variability in magnitude.

Numbers 33 and 34 form $\Sigma 1717$.

The rectangular coördinates were computed by the ordinary formulas

* Catalogue of the 3735 Circumpolar Stars observed at Redhill in 1854–55–56, by R. C. Carrington, p. xxv.

$$\left.\begin{aligned} X &= \cos a \sin \pi \operatorname{cosec} 1'' , \\ Y &= \sin a \sin \pi \operatorname{cosec} 1'' . \end{aligned}\right\} \qquad (13)$$

where $a$ and $\pi$ are the mean right ascension and polar distance of a star referred to the equator and equinox of the epoch $t_0$.

If $X_1$ and $Y_1$ are the rectangular coördinates for some other epoch $t$, they can be found from the following equations,

$$\left.\begin{aligned} X_1 &= X + \frac{dX}{dt}(t-t_0) + \tfrac{1}{2}\frac{d^2X}{dt^2}(t-t_0)^2 + \tfrac{1}{6}\frac{d^3X}{dt^3}(t-t_0)^3, \\ Y_1 &= Y + \frac{dY}{dt}(t-t_0) + \tfrac{1}{2}\frac{d^2Y}{dt^2}(t-t_0)^2 + \tfrac{1}{6}\frac{d^3Y}{dt^3}(t-t_0)^3. \end{aligned}\right\} \qquad (14)$$

The differential coefficients which are given in Table IX *seq.* were computed according to the formulas of Fabritius which may be found in several places, but are given in a particularly convenient form in Elkin's paper.*

The corresponding polar coördinates for the beginning of the year $t$, can be computed by the formulas

$$\left.\begin{aligned} \sin \pi_1 \cos a_1 &= X_1 \sin 1'', \\ \sin \pi_1 \sin a_1 &= Y_1 \sin 1''. \end{aligned}\right\} \qquad (15)$$

It is my intention later to determine the proper motions of the stars for which a sufficient number of accurate places exist. This will be deferred until a catalogue of the remaining eight plates has been made, after which the proper motion of the entire group will be deduced.

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* Loc. cit., p. 181.

TABLE VIII.—FINAL RESULTS.

| No. | Mg. | Right Asc. 1888.0. | N. P. D. | Declination, 1888.0. | X | Y | Number in B. D. M. | Number in Car. |
|---|---|---|---|---|---|---|---|---|
| | m | ° ′ ″ | ″ | ° ′ ″ | ″ | ″ | | |
| 1 | 10.3 | 1 46 5 | 3441.42 | 89 2 38.58 | +3439.63 | + 106.18 | 88.1 | 9 |
| 2 | 10.1 | 5 42 28 | 768.37 | 89 47 11.63 | + 764 56 | + 76.41 | 89.1 | *a* |
| 3 | 10.5 | 6 36 42 | 3156.86 | 89 7 23.14 | +3135.74 | + 363.46 | | |
| 4 | 9.7 | 24 5 50 | 2977.11 | 89 10 22.89 | +2717.56 | +1215.47 | 89.2 | 201 |
| 5 | 9.6 | 35 16 22 | 3431.68 | 89 2 48.32 | +2801.53 | +1981.60 | 88.10 | 303 |
| 6 | 10.8 | 35 59 0 | 3148.47 | 89 7 31.53 | +2547.61 | +1849.81 | | |
| 7 | 10.3 | 36 24 45 | 2238.21 | 89 22 41.79 | +1801.19 | +1328.56 | | |
| 8 | 10.3 | 38 12 49 | 2197.90 | 89 23 22.10 | +1726.88 | +1359.58 | 89.5 | 288 |
| 9 | 10.4 | 39 4 58 | 1633.24 | 89 32 46.76 | +1267.76 | +1029.65 | 89.4 | *c* |
| 10 | 10.3 | 41 38 13 | 3039.64 | 89 9 20.36 | +2271.65 | +2019.49 | 89.6 | 345 |
| 11 | 10.8 | 44 28 51 | 1427.80 | 89 36 12.20 | +1018.70 | +1000.41 | | |
| 12 | 10.1 | 57 50 56 | 2939.66 | 89 11 0.34 | +1564.30 | +2488.76 | 89.7 | 475 |
| 13 | 10.5 | 58 38 53 | 2652.79 | 89 15 47.21 | +1380.19 | +2265.38 | | |
| 14 | 9.7 | 63 16 44 | 3614.20 | 88 59 45.80 | +1625.04 | +3228.05 | 88.20 | 538 |
| 15 | 10.8 | 63 48 57 | 3211.01 | 89 6 28.99 | +1416.82 | +2881.38 | | |
| 16 | 10.7 | 67 39 59 | 3559.40 | 89 0 40.60 | +1352.50 | +3292.23 | | |
| 17 | 10.7 | 68 52 13 | 3276.83 | 89 5 23.17 | +1181.19 | +3056.39 | 89.8 | 579 |
| 18 | 10.5 | 70 48 23 | 1238.99 | 89 39 21.01 | + 407.33 | +1170.11 | | |
| 19 | 10.6 | 75 0 47 | 2360.12 | 89 20 39.88 | + 610.31 | +2279.78 | | |
| 20 | 10.8 | 83 4 41 | 2179.35 | 89 23 40.65 | + 262.64 | +2163.42 | | |
| 21 | 9.3 | 92 21 27 | 1887.19 | 89 28 32.81 | — 77.63 | +1885.57 | 89.9 | *d* |
| 22 | 10.3 | 98 0 17 | 3263.08 | 89 5 36.92 | — 454.38 | +3231.15 | 89.10 | 833 |
| 23 | 10.5 | 100 3 12 | 809.13 | 89 46 30.87 | — 141.24 | + 796.70 | | |
| 24 | 10.3 | 100 45 3 | 3444.34 | 89 2 35.66 | — 642.47 | +3383.73 | 89.11 | 862 |
| 25 | 10.0 | 120 19 9 | 1531.14 | 89 34 28.86 | — 772.94 | +1321.71 | 89.12 | *e* |
| 26 | 10.6 | 133 34 51 | 1298.28 | 89 38 21.72 | — 895.00 | + 940.47 | | |
| 27 | 10.3 | 154 5 20 | 2014.26 | 89 26 25.74 | —1811.75 | + 880.17 | 89.16 | *f* |
| 28 | 9.7 | 163 25 29 | 2301.17 | 89 21 38.83 | —2205.50 | + 656.45 | 89.17 | *g* |
| 29 | 10.3 | 171 19 4 | 2259.91 | 89 22 20.09 | —2233.97 | + 341.14 | 89.19 | |
| 30 | 9.9 | 172 39 5 | 1617.68 | 89 33 2.32 | —1604.37 | + 206.91 | 89.18 | *h* |
| 31 | 10.3 | 180 26 32 | 2687.34 | 89 15 12.66 | —2687.18 | — 20.74 | 89.20 | |
| 32 | 10.5 | 191 13 35 | 2067.17 | 89 25 32.83 | —2027.59 | — 402.44 | | |
| 33 | 10.6 | 191 16 49 | 2540.66 | 89 17 39.34 | —2491.51 | — 496.96 | | |
| 34 | 9.7 | 191 22 6 | 2547.86 | 89 17 32.14 | —2497.81 | — 502.21 | 89.21 | *i* |
| 35 | 10.3 | 195 35 20 | 2274.06 | 89 22 5.94 | —2190.36 | — 611.10 | 89.23 | *k* |

TABLE VIII.—(*Concluded.*) FINAL RESULTS.

| No. | Mg. | Right Asc. 1888.0. | N. P. D. | Declination, 1888.0. | X | Y | Number in B. D. M. | Number in Cat. |
|---|---|---|---|---|---|---|---|---|
| | m | ° ′ ″ | ″ | ° ′ ″ | ″ | ″ | | |
| 36 | 10.3 | 196 52 6 | 861.96 | 89 45 38.04 | — 824.87 | — 250.12 | 89.26 | *m* |
| 37 | 11.0 | 201 45 8 | 811.09 | 89 46 28.91 | — 753.34 | — 300.58 | | |
| 38 | 10.1 | 211 36 12 | 1658.03 | 89 32 21.97 | —1412.12 | — 868.86 | 89 25 | *l* |
| 39 | 10.6 | 215 48 38 | 2185.64 | 89 23 34.36 | —1772.42 | —1278.81 | | |
| 40 | 10.7 | 217 21 47 | 2425.88 | 89 19 34.12 | —1928.06 | —1472.14 | | |
| 41 | 10.5 | 229 24 54 | 1374.18 | 89 37 5.82 | — 894.00 | —1043.60 | | |
| 42 | 10.3 | 237 21 25 | 1153.85 | 89 40 46.15 | — 622.39 | — 971.59 | 89.29 | *o* |
| 43 | 10.6 | 238 14 32 | 1912.82 | 89 28 7.18 | —1006.76 | —1626.41 | | |
| 44 | 10.6 | 244 34 34 | 2811.15 | 89 13 8.85 | —1206.82 | —2538.83 | | |
| 45 | 10.0 | 254 55 12 | 731.86 | 89 47 48.14 | — 190.41 | — 706.66 | 89.35 | *r* |
| 46 | 10.3 | 262 58 32 | 1230.43 | 89 39 29.57 | — 150.47 | —1221.19 | 89.31 | *p* |
| 47 | 10.5 | 263 51 20 | 3212.55 | 89 6 27.45 | — 343.84 | —3193.96 | | |
| 48 | 11.0 | 265 29 32 | 328.65 | 89 54 31.35 | — 25.83 | — 327.63 | | |
| 49 | 10.1 | 266 12 37 | 340.07 | 89 54 19.93 | — 22.48 | — 339.33 | 89.37 | *t* |
| 50 | 10.0 | 268 28 50 | 2977.90 | 89 10 22.10 | — 78.96 | —2976.75 | 89.30 | 2818 |
| 51 | 10.3 | 277 1 58 | 1758.92 | 89 30 41.08 | + 215.35 | —1745.66 | 89.33 | *g* |
| 52 | 10.3 | 281 36 19 | 2621.92 | 89 16 18.08 | + 527.43 | —2568.24 | 89.32 | 2977 |
| 53 | 10.6 | 281 36 14 | 1211.82 | 89 39 48.18 | + 243.75 | —1187.04 | | |
| 54 | 9.9 | 285 42 0 | 2430.61 | 89 19 29.39 | + 657.71 | —2339.87 | 89.34 | 3023 |
| 55 | 11.0 | 291 37 40 | 502.49 | 89 51 37.51 | + 185.20 | — 467.11 | | |
| 56 | 10.6 | 292 6 52 | 1593.48 | 89 33 26.52 | + 599.87 | —1476.24 | | |
| 57 | 10.8 | 293 39 27 | 1055.52 | 89 42 24.48 | + 423.54 | — 966.81 | | |
| 58 | 10.3 | 297 15 9 | 1291.24 | 89 38 28.76 | + 591.27 | —1147.90 | 89.36 | *s* |
| 59 | 10.9 | 307 22 0 | 1541.84 | 89 34 18.16 | + 935.76 | —1225.39 | | |
| 60 | 10.6 | 311 8 28 | 851.61 | 89 45 48.39 | + 560.29 | — 641.34 | | |
| 61 | 10.4 | 314 53 27 | 1581.52 | 89 33 38.48 | +1116.16 | —1120.42 | | |
| 62 | 10.4 | 315 28 36 | 1353.56 | 89 37 26.44 | + 965.03 | — 949.11 | | |
| 63 | 10.3 | 317 46 13 | 3252.86 | 89 5 47.14 | +2408.50 | —2186.17 | 88.126 | |
| 64 | 10.4 | 322 7 13 | 1186.00 | 89 40 14.00 | + 936.11 | — 728.21 | | |
| 65 | 9.7 | 348 36 18 | 2904.70 | 89 11 35.30 | +2847.35 | — 573.87 | 89.38 | 3601 |

TABLE IX.—PRECESSION.

| Star. | $dy$ | $dx$ | 100 $d^2y$ | 100 $d^2x$ | 10,000 $d^3y$ | 10,000 $d^3x$ |
|---|---|---|---|---|---|---|
| | ″ | ″ | ″ | ″ | ″ | ″ |
| 1 | +0.7686 | —20.0740 | —0.4480 | —0.0118 | —0.0008 | +0.0119 |
| 2 | +0.1708 | —20.0701 | —0.4484 | +0.0041 | —0.0005 | +0.0119 |
| 3 | +0.7006 | —20.1320 | —0.4494 | —0.0100 | —0.0008 | +0.0119 |
| 4 | +0.6072 | —20.3226 | —0.4537 | —0.0077 | —0.0007 | +0.0121 |
| 5 | +0.6260 | —20.4931 | —0.4575 | —0.0083 | —0.0007 | +0.0122 |
| 6 | +0.5692 | —20.4641 | —0.4568 | —0.0067 | —0.0007 | +0.0121 |
| 7 | +0.4025 | —20.3487 | —0.4544 | —0.0023 | —0.0006 | +0.0121 |
| 8 | +0.3859 | —20.3558 | —0.4546 | —0.0018 | —0.0006 | +0.0121 |
| 9 | +0.2833 | —20.2826 | —0.4529 | +0.0010 | —0.0005 | +0.0120 |
| 10 | +0.5076 | —20.5021 | —0.4578 | —0.0052 | —0.0007 | +0.0122 |
| 11 | +0.2276 | —20.2761 | —0.4529 | +0.0025 | —0.0005 | +0.0120 |
| 12 | —0.3495 | —20.6072 | —0.4602 | —0.0010 | —0.0006 | +0.0122 |
| 13 | +0 3084 | —20.5576 | —0.4591 | +0.0001 | —0.0006 | +0.0122 |
| 14 | +0.3631 | —20.7713 | —0.4639 | —0.0014 | —0.0006 | +0.0123 |
| 15 | +0.3166 | —20.6945 | —0.4622 | —0.0002 | —0.0006 | +0.0123 |
| 16 | +0.3022 | —20.7858 | —0.4642 | +0.0002 | —0.0005 | +0.0123 |
| 17 | +0.2639 | —20.7335 | —0.4631 | +0.0012 | —0.0005 | +0.0123 |
| 18 | +0.0910 | —20.3141 | —0.4538 | +0.0060 | —0.0004 | +0.0121 |
| 19 | +0.1364 | —20.5612 | —0.4593 | +0.0047 | —0.0004 | +0.0122 |
| 20 | +0.0587 | —20.5354 | —0 4588 | +0.0067 | —0.0004 | +0.0122 |
| 21 | —0.0173 | —20.4736 | —0.4574 | +0.0088 | —0.0003 | +0.0121 |
| 22 | —0.1015 | —20.7726 | —0.4642 | +0.0109 | —0.0003 | +0.0123 |
| 23 | —0.0316 | —20.2310 | —0.4521 | +0.0093 | —0.0003 | +0.0120 |
| 24 | —0.1436 | —20.8064 | —0.4650 | +0.0119 | —0.0003 | +0.0123 |
| 25 | —0.1737 | —20.3479 | —0.4547 | +0.0130 | —0.0002 | +0.0121 |
| 26 | —0.2000 | —20.2628 | —0.4528 | +0.0138 | —0.0002 | +0.0120 |
| 27 | —0.4048 | —20.2489 | —0.4526 | +0.0193 | —0.0001 | +0.0120 |
| 28 | —0.4928 | —20.1984 | —0.4516 | +0.0216 | —0.0001 | +0.0120 |
| 29 | —0.4992 | —20.1281 | —0.4500 | +0.0219 | —0 0000 | +0.0119 |
| 30 | —0.3585 | —20.0987 | —0.4492 | +0.0181 | —0.0001 | +0.0119 |
| 31 | —0.6004 | —20.0468 | —0.4483 | +0.0246 | —0.0000 | +0.0119 |
| 32 | —0.4530 | —19.9622 | —0.4463 | +0.0206 | —0.0001 | +0.0119 |
| 33 | —0.5567 | —19.9406 | —0.4458 | +0.0235 | —0.0000 | +0.0118 |
| 34 | —0.5581 | —19.9394 | —0.4458 | +0.0235 | —0.0000 | +0.0118 |
| 35 | —0.4894 | —19.9154 | —0.4453 | +0.0217 | —0.0001 | +0.0118 |

TABLE IX.—(*Concluded.*) PRECESSION.

| Star. | $dy$ | $dx$ | 100 $d^2y$ | 100 $d^2x$ | 10,000 $d^3y$ | 10,000 $d^3x$ |
|---|---|---|---|---|---|---|
| | " | " | " | " | " | " |
| 36 | —0.1843 | —19.9970 | —0.4470 | +0.0135 | —0.0002 | +0.0119 |
| 37 | —0.1683 | —19.9858 | —0.4467 | +0.0131 | —0.0003 | +0.0119 |
| 38 | —0.3155 | —19.8584 | —0.4439 | +0.0171 | —0.0002 | +0.0118 |
| 39 | —0.3960 | —19.7663 | —0.4418 | +0.0193 | —0.0001 | +0.0117 |
| 40 | —0.4308 | —19.7228 | —0.4409 | +0.0202 | —0.0001 | +0.0117 |
| 41 | —0.1998 | —19.8194 | —0.4429 | +0.0140 | —0.0002 | +0.0118 |
| 42 | —0.1391 | —19.8357 | —0.4433 | +0.0124 | —0.0003 | +0.0118 |
| 43 | —0.2249 | —19.6888 | —0.4400 | +0.0148 | —0.0002 | +0.0117 |
| 44 | —0.2696 | —19.4840 | —0.4355 | +0.0162 | —0.0002 | +0.0116 |
| 45 | —0.0425 | —19.8951 | —0.4446 | +0.0098 | —0.0003 | +0.0118 |
| 46 | —0.0336 | —19.7798 | —0.4419 | +0.0097 | —0.0003 | +0.0117 |
| 47 | —0.0768 | —19.3370 | —0.4321 | +0.0110 | —0.0003 | +0.0115 |
| 48 | —0.0058 | —19.9799 | —0.4465 | +0.0088 | —0.0004 | +0.0119 |
| 49 | —0.0050 | —19.9773 | —0.4464 | +0.0087 | —0.0004 | +0.0119 |
| 50 | —0.0176 | —19.3859 | —0.4331 | +0.0095 | —0.0003 | +0.0115 |
| 51 | +0.0481 | —19.6624 | —0.4393 | +0.0075 | —0.0004 | +0.0117 |
| 52 | +0.1178 | —19.4777 | —0.4351 | +0.0059 | —0.0004 | +0.0116 |
| 53 | +0.0545 | —19.7876 | —0.4421 | +0.0074 | —0.0004 | +0.0117 |
| 54 | +0.1470 | —19.5289 | —0.4362 | +0.0050 | —0.0005 | +0.0116 |
| 55 | +0.0414 | —19.9487 | —0.4458 | +0.0076 | —0.0004 | +0.0118 |
| 56 | +0.1340 | —19.7226 | —0.4405 | +0.0052 | —0.0004 | +0.0117 |
| 57 | +0.0946 | —19.8368 | —0.4432 | +0.0062 | —0.0004 | +0.0118 |
| 58 | +0.1321 | —19.7962 | —0.4422 | +0.0053 | —0.0004 | +0.0117 |
| 59 | +0.2091 | —19.7788 | —0.4418 | +0.0032 | —0.0004 | +0.0117 |
| 60 | +0.1252 | —19.9096 | —0.4448 | +0.0054 | —0.0004 | +0.0118 |
| 61 | +0.2494 | —19.8022 | —0.4422 | +0.0022 | —0.0005 | +0.0118 |
| 62 | +0.2156 | —19.8406 | —0.4432 | +0.0030 | —0.0005 | +0.0118 |
| 63 | +0.5382 | —19.5621 | —0.4368 | —0.0054 | —0.0007 | +0.0116 |
| 64 | +0.2092 | —19.8901 | —0.4443 | +0.0031 | —0.0005 | +0.0118 |
| 65 | +0.6362 | —19.9229 | —0.4447 | —0.0082 | —0.0007 | +0.0118 |

## X.

## Optical Distortion.

The subject of optical distortion has been approached from both theoretical and empirical standpoints. Scheiner, in his "Photographie der Gestirne"* gives the theoretical aspect, and treats the subject under the heads of regular and irregular distortion. Regular distortion is due to the fact that a spherical surface is projected on the flat photographic plate, and its effect can be eliminated by formulas of reduction such as equations (1) of this paper. Irregular distortion is a peculiar property of each telescope, and depends upon the curvature of the lenses and the indices of refraction, *i. e.*, the constants of the system as they are usually understood in optics. Its effect is to change the shape of the star image by elongating it radially, and under certain circumstances to displace the center of density.

Steinheil† has computed the effect of the Konigsberg heliometer objective on a cylinder of rays passing through the lens, supposing the photographic plate to be at different distances from the objective. A star image 48′ from the principal axis becomes somewhat comet shaped, though it is still symmetrical with respect to a line drawn through it from the center of the plate. The consequence is, that there would be difficulty in deciding how to bisect the image, and the measured positions may not represent the positions of the stars in the sky. However, the heliometer is corrected for visual rays only. In the case of a photographic objective properly corrected, the image would become elliptical in shape, would be symmetrical in two directions, and the centers of density and figure would coincide, so that the difficulty of bisection arising from this cause is removed.

The subject has been more extensively treated from the empirical standpoint. Prof. H. H. Turner,‡ has published a paper on the errors of star places due to optical distortion in which he shows that the plates of the Oxford Astrophotographic telescope are free from distortion by the examination of forty plates, each containing about thirty stars. From the smallness of the resid-

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* Pp. 23–25.

† Scheiner, pp, 25–27, see footnote, p. 25.

‡ Mon. Not., R. A. S. Vol. LIX, p. 438.

uals he deduces the fact that there is no optical distortion.* In the second part of his paper, Prof. Turner discusses the possible distortion of a different kind of glass, namely the Bruce photographic doublet, the property of the Harvard College Observatory, and now at the Arequipa station. Through Prof. Pickering, a glass positive from Arequipa was secured, and two contact negatives were made of overlapping portions of the positive on the regulation Oxford plates. Two other Oxford plates were exposed to the same region in the sky, and were compared with the contact negatives. The residuals obtained by this process† were too large and too systematic to receive the treatment accorded to the residuals from the Oxford plates. After some further examination it was decided that the discrepancies were due to the curvature of the plates used in making the photographic copies, to the want of complete contact and to obliqueness in the illumination.‡ His final conclusion is that there is no optical distortion over an area 4° from the center of the Harvard plate.

In a later number§ of the same periodical appears an article by Capt. Hills describing another method of finding distortion employed by him. He photographed star trails on a plate, and measuring the actual differences in curvature between them, compared them with the computed differences. In applying this method to a Zeiss doublet of $377^{mm}$ focal length, he found that there was no measurable distortion over a field of 9° from the center.

The distortion caused by the Helsingfors telescope with which the polar photographs were taken, and which is the general subject of this section, has already been investigated by Professor Donner, the Director of the Observatory.¶ Briefly described, his method was to compare the measured coördinates of a group of stars with the coördinates computed from their right ascensions and declinations, which had been accurately determined by some other method than the photographic. Accordingly he selected a group of stars contained in a circular issued by Dr. Gill for this very purpose. This group formed a portion of the Victoria comparison stars used by Gill in his heliometer triangulation for solar

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* Loc cit., p. 443.

† Loc. cit., p. 450.

‡ Loc. cit., p. 465.

§ Mon. Not., R. A. S., Vol. LIX, p. 564.

¶ See footnote, p. 5, this paper.

parallax, and their places had been determined with great precision. This group was photographed on six plates when near the meridian, with the telescope in the two positions, east and west of the pier. Without going into the details of his method, it may be mentioned that there are nine stars on each plate and that the rectangular coördinates were measured from the star nearest the center. The coördinates were corrected for refraction, aberration and division errors of the reseau, and were then compared with the coördinates of the same stars computed from the places furnished by Gill. With the differences O–C as the absolute terms in the equations

$$0 = n_y + k_y + py - rx,$$

$$0 = n_x + k_x + px + ry,$$

the plate constants were computed in the usual way. In solving these equations Donner used the $x$-equations and the $y$-equations both together and separately, with interesting results as regards the scale value. This was found to be

$$p_y = -0.002134,$$

$$p_x = -0.002036,$$

$$p_{xy} = -0.002072.$$

Thus there appears to be a uniform difference in the two scale values of

$$p_y - p_x = -0.000098,$$

which is equivalent to 0′.0059 at a distance of 60′ from the center of the plate. This difference seems to indicate that the object glass flattens the field, making the $x$-coördinates longer in proportion than the $y$-coördinates, and might be taken as an indication of distortion. Donner himself suggests several other explanations, but seems to think it is especially due to small errors in the star places.

There is still another explanation, which latter became available, and which was suggested to me by Professor Jacoby. The corrections for refraction were calculated according to Kapteyn's formulas, which were published by him in the Bulletin of the Permanent Committee,* and are also to be found in Scheiner†.

* Bull. Com. Perm., T. I, p. 101.

† Loc. cit., p. 140.

In a latter number * Kapteyn published a third term for the refraction in $x$, which is

$$-k \tan \delta \cot (\delta + N)x.$$

Wishing to learn the effect of this slight additional term, I made a new reduction of Exposure I, Plate 1, Sept. 12, 1892. On this plate $p_y$ is $-0.002152$ and $p_x$ — is 0.002052, showing the average difference above referred to. The absolute terms arising from the $x$-coördinates are all increased numerically as will be seen by comparing the following set of numbers:

| $n_1$(old). | $n_1$(new). |
|---|---|
| —.1292 | —.1354 |
| —.1019 | —.1069 |
| —.0734 | —.0772 |
| —.0396 | —.0414 |
| —.0296 | —.0311 |
| —.0000 | —.0000 |
| +.0622 | +.0666 |
| +.0967 | +.1012 |
| +.1285 | +.1346 |

A second solution of the equations with the new absolute terms leads to the new value of

$$p_x = -0.002164.$$

This value accords very well with the value of $p_y$, and hence, as far as the difference in scale values is concerned, there seems to be no evidence of distortion.

The expression which Donner himself used for this displacement was

$$as + bs^2 + cs^3 + \ldots,$$

where $s$ is the distance from the center, and $a$, $b$ and $c$ are constants to be determined from the equations. On account of the form of the terms, $a$ cannot be separated from the scale value, hence $b$ is the only constant obtained. The six plates in the Victoria region give for $b$ the mean value,

$$b = -0.0000010,$$

and the reduction of the two plates of the Pleiades furnishes for $b$ the mean value,

$$b = +0.0000021.$$

* Corrections de Réfraction et d'Aberration. Bull. Com. Perm., Tom. III, 1896. (Not yet published.)

Calling attention to the small magnitude of these quantities and their difference in sign, Donner concludes* that the constant $b$ of the term $bs^2$ in the expression for the distortion does not exceed a few units in the seventh decimal place, and since the corrections which might result are for all practical purposes confused with the corrections for the scale value of the plate, we can in practice neglect the distortion of the field of the objective.

It will thus be seen that Donner's and Turner's methods are alike in comparing the measured coördinates with the coördinates obtained from other observations, for which purpose Donner used heliometer measurements and Turner, meridian observations.

Another similar investigation making use of the same stars in Gill's circular was carried out at the Observatory of San Fernando†, one of the observatories participating in the astrophotographic survey. The same formula

$$c = \rho s + bs^2$$

was employed, and its effect was found to be negligible. The details of the method of research are not published in the Bulletin.

It was stated in the introduction to this paper that one of Prof. Jacoby's objects in having a group of polar plates taken was to investigate the distortion of the Helsingfors objective by a new method which did not involve, except indirectly, any previous determination of the star places. His plan was to have the same group of stars photographed with the telescope in different positions with regard to the pier, so that the object glass would be situated differently each time with regard to the stars in the group. Two different positions could be obtained by taking a group near the meridian, and reversing the telescope between the two exposures which method was indeed followed by Donner, though for another purpose. But the selection of the group about the pole made it possible to turn the telescope through any angle desired after each exposure. On reference to Table I, page 3, it will be seen that the right ascensions of the pole were

$$19^h24^m, \quad 22^h24^m, \quad 1^h24^m, \quad 4^h24^m;$$

thus differing by $45^\circ$. Strictly speaking, the pole has every right

* Loc. cit., p. 65.

† Bulletin du Comité Permanent, 1896; Procès-Verbaux, p. 52.

ascension, but those just quoted belong to the hour circles which are perpendicular to the $X$ axes on the different plates.

As before stated, the usual effect of optical distortion is to displace the center of density of a star image. If the displacement is only radial, it will change the distance of the star from the center of the plate, but will not change its position angle, but if, in addition to the radial displacement, there is a displacement in any other direction due to some special peculiarity of the object glass, such as would be suggested by a different scale value in two perpendicular directions, the combination of the two would alter both the position angle and distance of the star, and this effect might be different over different parts of the plate. A displacement of this character would not be detected by Donner's method, which assumes that the distortion is the same along different radii of the objective. Furthermore, if the plate holder always keeps the same position relative to the object glass, which is usually the case with telescopes employed only for photographic purposes, and if the telescope is changed in position, then the same star appearing on different portions of the plates, will be displaced differently and a proper comparison of the right ascensions and polar distances of a large number of stars, obtained independently from the different plates would give the effect of this kind of distortion. This method of procedure was followed with the polar plates taken at Helsingfors.

From the Table of Mean Results, forty-two stars (there marked with an asterisk), which appear on all four plates were selected on the basis of their distribution in right ascension and polar distance. They were divided into zones, and about the same number of stars was selected from each zone, as will be seen on referring to the figures given below.

| | | | |
|---|---|---|---|
| I | 0″–1300″ | 2, 23, 42, 45, 46, 49, 53, 58, 60, 64, | Total 10. |
| II | 1300″–1800″ | 9, 11, 25, 26, 30, 41, 56, 62, | " 8. |
| III | 1800″–2400″ | 8, 19, 21, 28, 29, 32, 35, 39, 43, | " 9. |
| IV | 2400″–3000″ | 4, 12, 38, 44, 50, 52, 54, 65, | " 8. |
| V | 3000″–3600″ | 1, 3, 10, 14, 24, 47, 63, | " 7. |

By means of these forty-two stars the four plates were reduced to a common standard as follows: A new set of corrections to the constants of reduction for each plate was obtained by a least squares solution according to equations (2) using the above forty-two stars for this purpose, and taking for the numerical terms the residuals found in the columns headed $\Delta\alpha \sin \pi$ and $\Delta\pi$ of Table VII. For example the following equations are obtained from stars 1 and 2, plate I.

$$\begin{array}{lr} & v \\ -54d\xi - 25d\eta + 58dA' - 05 = 0, & +07\,; \\ -25d\xi + 54d\eta + 58d\omega \;\; + 22 = 0, & +08\,; \\ -52d\xi - 30d\eta + 13dA' \;\; - 24 = 0, & -18\,; \\ -30d\xi + 52d\eta + 13d\omega \;\; - 02 = 0, & -04. \end{array}$$

The new residuals obtained by substituting the $d\xi$, $d\eta$, $d\omega$ and $dA'$ in the equations just described, involve only the systematic errors due to distortion. They are given in the above equations under the heading "$v$." In general the effect of this new reduction was to decrease numerically the residuals to a small extent only. In fact it will be noticed in referring to the residuals in Table VII, that they are fairly well distributed both in magnitude and sign, and from this it was inferred beforehand that the corrections to $\xi$, $\eta$, $A$ and $\omega$ would be small and would not materially change the characters of the residuals, as indeed turned out to be the case.

A satisfactory agreement as regards the scale value appeared in the course of this reduction, and in some sense it is a test of the method. It can be quite easily perceived by an examination of the following figures which give the old and new values.

| Plate. | $\omega$ Table VI. | $d\omega$ | $\omega$ (new). |
|---|---|---|---|
| I $\frac{1}{2}(B+M)$ | 59″.8590 | −0″.0026 | 59″.8564 |
| II $\frac{1}{2}(B+M)$ | .8566 | −0 .0010 | .8556 |
| III $\frac{1}{2}(B+M)$ | .8528 | +0 .0021 | .8549 |
| IV $\frac{1}{2}(B+M)$ | .8561 | +0 .0015 | .8576 |
| Mean | 59″.8561 ± 0″.0017 | | 59″.8561 ± 0″.0008 |

The following table gives the new residuals for the forty-two stars:

TABLE X.

| Star. | $B$ | $\Delta\alpha \sin \pi$ | $p$ | $\Delta\pi$ |
|---|---|---|---|---|
| | ° | ″ | mm | ″ |
| 1 | 294 | +.07 | 58 | +.08 |
| | 249 | —.31 | | —.35 |
| | 340 | +.54 | | —.20 |
| | 207 | —.34 | | +.45 |
| 3 | 299 | —.16 | 53 | +.08 |
| | 254 | —.11 | | +.07 |
| | 344 | +.04 | | —.16 |
| | 211 | +.23 | | +.02 |
| 4 | 317 | —.04 | 50 | —.26 |
| | 272 | —.18 | | —.32 |
| | 2 | —.08 | | +.09 |
| | 229 | +.26 | | +.49 |
| 8 | 331 | +.02 | 37 | +.15 |
| | 286 | +.04 | | —.05 |
| | 16 | +.06 | | +.22 |
| | 243 | —.15 | | —.36 |
| 9 | 332 | +.33 | 28 | +.32 |
| | 287 | —.14 | | —.09 |
| | 17 | +.17 | | +.13 |
| | 244 | —.37 | | —.35 |
| 10 | 334 | —.04 | 51 | —.01 |
| | 289 | —.26 | | +.15 |
| | 19 | +.00 | | +.10 |
| | 246 | +.26 | | —.26 |
| 11 | 337 | —.20 | 24 | —.23 |
| | 292 | —.28 | | +.35 |
| | 22 | —.10 | | —.42 |
| | 250 | +.56 | | +.26 |
| 12 | 350 | +.14 | 49 | +.17 |
| | 305 | —.16 | | +.32 |
| | 36 | +.23 | | —.17 |
| | 263 | —.24 | | —.32 |
| 14 | 356 | +.10 | 61 | —.13 |
| | 311 | +.15 | | —.07 |
| | 41 | +.14 | | +.09 |
| | 268 | —.36 | | +.11 |
| 19 | 8 | —.08 | 40 | —.12 |
| | 322 | +.06 | | —.03 |
| | 53 | +.01 | | —.13 |
| | 280 | —.01 | | +.25 |
| 21 | 25 | —.03 | 32 | +.29 |
| | 340 | +.18 | | —.11 |
| | 70 | —.25 | | —.18 |
| | 297 | +.10 | | —.01 |
| 24 | 33 | —.15 | 58 | +.07 |
| | 348 | +.01 | | +.33 |
| | 78 | —.09 | | —.40 |
| | 305 | +.22 | | —.03 |
| 25 | 52 | +.13 | 26 | —.26 |
| | 7 | +.23 | | +.16 |
| | 97 | +.04 | | +.07 |
| | 324 | —.38 | | +.03 |
| 26 | 65 | +.03 | 22 | —.07 |
| | 20 | +.16 | | +.26 |
| | 110 | —.15 | | —.37 |
| | 338 | —.04 | | +.17 |

TABLE X.—(*Continued*).

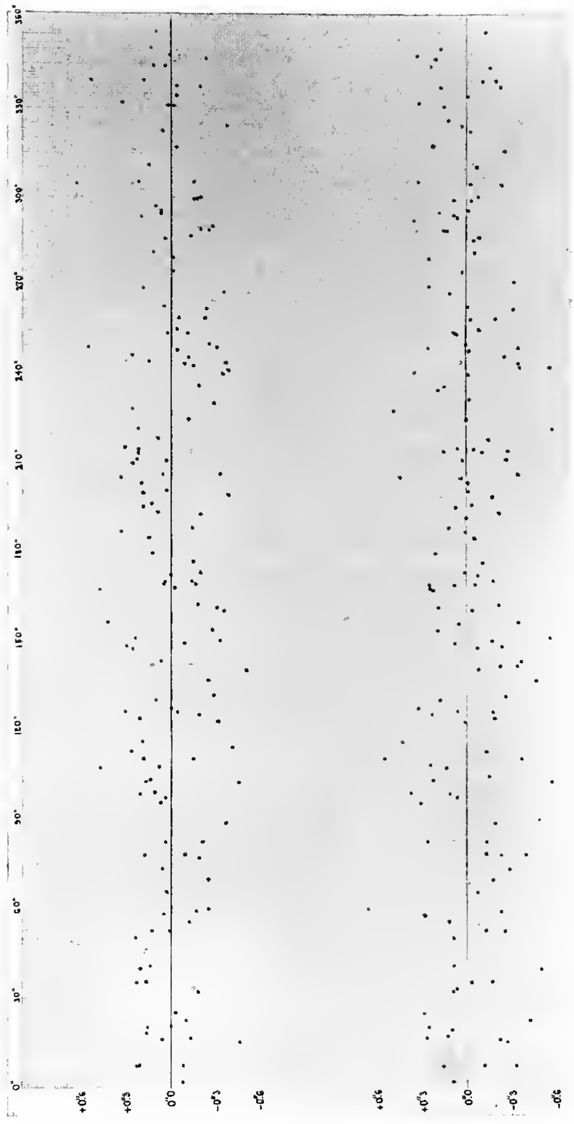
| Star. | *B* | Δα sin π | *p* | Δπ |
|---|---|---|---|---|
| | ° | ″ | mm | ″ |
| 28 | 95 | +.07 | 38 | +.32 |
| | 50 | +.24 | | +.09 |
| | 140 | —.51 | | —.08 |
| | 8 | +.21 | | —.33 |
| 29 | 103 | +.14 | 38 | +.23 |
| | 58 | +.05 | | +.29 |
| | 148 | +.30 | | —.24 |
| | 15 | —.46 | | —.27 |
| 30 | 104 | +.41 | 27 | —.15 |
| | 59 | —.17 | | —.23 |
| | 150 | —.09 | | +.08 |
| | 16 | —.13 | | +.27 |
| 32 | 123 | —.32 | 34 | +.01 |
| | 78 | +.18 | | —.23 |
| | 168 | —.02 | | +.25 |
| | 35 | +.17 | | —.03 |
| 35 | 128 | +.00 | 38 | +.33 |
| | 82 | —.21 | | +.27 |
| | 173 | +.00 | | —.08 |
| | 40 | +.21 | | —.50 |
| 38 | 144 | +.07 | 27 | —.37 |
| | 98 | +.21 | | +.12 |
| | 189 | —.14 | | +.12 |
| | 56 | —.12 | | +.12 |
| 39 | 148 | +.26 | 36 | —.07 |
| | 103 | +.17 | | —.57 |
| | 193 | —.20 | | +.00 |
| | 60 | —.25 | | +.67 |
| 41 | 161 | —.31 | 23 | +.19 |
| | 116 | +.19 | | +.44 |
| | 206 | +.06 | | —.35 |
| | 73 | +.06 | | —.29 |
| 42 | 169 | +.05 | 19 | —.06 |
| | 124 | +.21 | | —.19 |
| | 214 | +.22 | | —.28 |
| | 81 | —.46 | | +.53 |
| 43 | 170 | —.14 | 32 | —.18 |
| | 125 | —.19 | | +.24 |
| | 216 | +.31 | | +.06 |
| | 82 | +.04 | | —.13 |
| 44 | 177 | —.15 | 47 | —.11 |
| | 132 | +.29 | | —.26 |
| | 222 | +.22 | | +.58 |
| | 89 | —.37 | | —.19 |
| 45 | 187 | +.34 | 12 | +.01 |
| | 142 | —.13 | | —.34 |
| | 232 | —.32 | | —.02 |
| | 99 | +.11 | | +.38 |
| 46 | 195 | +.19 | 20 | +.07 |
| | 150 | —.33 | | —.17 |
| | 240 | —.35 | | —.01 |
| | 107 | +.48 | | +.14 |
| 47 | 196 | +.13 | 53 | —.04 |
| | 151 | +.24 | | —.56 |
| | 241 | —.39 | | +.35 |
| | 108 | +.06 | | +.25 |

TABLE X.—(*Concluded*).

| Star. | B | Δα sin π | p | Δπ |
|---|---|---|---|---|
| | ° | ″ | mm | ″ |
| 49 | 199 | +.19 | 5 | —.18 |
| | 154 | —.28 | | +.20 |
| | 244 | —.10 | | —.56 |
| | 111 | +.19 | | +.56 |
| 50 | 201 | +.03 | 49 | —.01 |
| | 156 | —.16 | | +.05 |
| | 246 | —.12 | | +.12 |
| | 113 | +.27 | | —.13 |
| 52 | 214 | —.13 | 43 | —.11 |
| | 169 | +.04 | | +.08 |
| | 259 | —.23 | | —.03 |
| | 126 | +.31 | | +.06 |
| 53 | 214 | +.22 | 20 | +.15 |
| | 169 | —.16 | | +.25 |
| | 259 | —.05 | | —.20 |
| | 126 | —.04 | | —.18 |
| 54 | 218 | +.09 | 40 | —.15 |
| | 173 | —.20 | | +.01 |
| | 263 | +.05 | | —.01 |
| | 130 | +.10 | | +.18 |
| 56 | 225 | —.12 | 26 | +.00 |
| | 180 | +.13 | | +.21 |
| | 270 | +.19 | | +.25 |
| | 137 | —.25 | | —.47 |
| 58 | 230 | —.29 | 21 | +.19 |
| | 185 | +.15 | | —.05 |
| | 275 | —.01 | | +.03 |
| | 142 | +.13 | | —.23 |
| 60 | 245 | +.15 | 14 | +.04 |
| | 199 | —.39 | | +.17 |
| | 290 | —.20 | | +.13 |
| | 157 | +.43 | | —.35 |
| 62 | 249 | —.04 | 22 | —.02 |
| | 203 | +.20 | | —.01 |
| | 294 | +.20 | | +.06 |
| | 161 | —.36 | | —.04 |
| 63 | 251 | —.26 | 54 | +.00 |
| | 205 | +.34 | | +.04 |
| | 296 | +.07 | | +.18 |
| | 163 | —.18 | | —.22 |
| 64 | 255 | —.04 | 20 | —.09 |
| | 210 | +.26 | | —.05 |
| | 300 | —.20 | | —.08 |
| | 167 | +.48 | | +.23 |
| 65 | 281 | +.11 | 48 | —.06 |
| | 236 | —.19 | | +.15 |
| | 326 | —.02 | | +.11 |
| | 193 | +.09 | | —.22 |
| 2 | 300 | —.18 | 13 | —.04 |
| | 254 | +.02 | | +.08 |
| | 345 | +.12 | | +.24 |
| | 212 | +.03 | | —.27 |
| 23 | 32 | —.19 | 14 | +.09 |
| | 347 | —.24 | | +.21 |
| | 77 | —.19 | | —.07 |
| | 304 | +.64 | | —.24 |

To facilitate a comparison of the residuals they were plotted as ordinates of points whose abscissas were the position angles $B$; those for $\Delta\alpha \sin \pi$ forming the upper part of the diagram and those for $\Delta\pi$ the lower. Thus the residuals from stars lying on the same section of the plates were grouped together. An examination of the accompanying diagram shows that there is no decided tendency on the part of the residuals toward a grouping such as would indicate the presence of distortion. Indeed two stars close together on the plate have quite different residuals, *vide* 32 and 35. The plot of the points representing the residuals before the least squares reduction compared with the plot of the residuals after reduction, shows simply that the latter are more crowded together toward the axis of abscissas than the former, but there is no especial distribution.

The conclusion is therefore reached that this method of comparison does not show any indication of distortion in the field of the objective. This result is one that was to be expected from Donner's investigation, as well as from optical considerations. The tests to which the instrument maker subjects his lens before it leaves his hands as a finished product are severe and delicate. An elevation equal to the ~~one~~ two hundred thousandth part of an wave-length of light can be detected, and the tests for striæ and variations in density are equally exacting. The radial distortion is a natural consequence of the curve of the glass, but a displacement in position angle or a deformation such as was suggested by the discrepant scale values in the two coördinate directions, must in all probability be due to something which happens to a glass after it leaves the hands of the maker, as for instance, a strain due to the position of the lens in its cell. Hence, the general conclusion is reached, that so far as the Helsingfors telescope is concerned, there is no evidence of optical distortion, and that in this respect its performance is most satisfactory.



Plot of Residuals from Table X.